MT

2018 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 6

Time : 2 Hours **(Model Answer Paper)** **Max. Marks : 40**

A.1. **(A) Solve the following : (Any 4)** **1**

(1) Perimeter of a parallelogram = 2 (sum of lengths of adjacent sides) ½

= 2 (3 + 4) = 14 cm ½

$\therefore$ Perimeter of a parallelogram is 14 cm

(2) Longest chord of a circle is Diameter ½

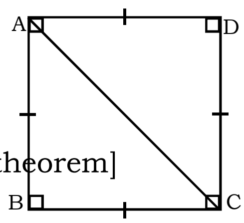
Diameter = 2 × radius

= 2 × 8

= 16

$\therefore$ Length of longest chord of a circle is 16 cm. ½

(3) Let □ABCD be a square



AB = BC = 5 cm

In ΔABC,

$\angle ABC = 90^\circ$ (Angle of a square)

$AC^2 = AB^2 + BC^2$ [Pythagoras theorem] ½

$= 5^2 + 5^2$

$= 25 + 25$

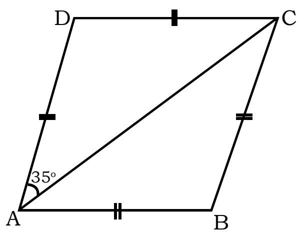
$\therefore AC^2 = 50$

$\therefore AC = \sqrt{50} = \sqrt{25 \times 2}$

$\therefore AC = 5\sqrt{2}$

Length of the diagonal is $5\sqrt{2}$ cm ½

(4)



□ABCD is a rhombus

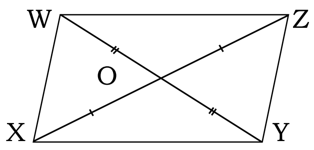
In ΔDAC,
side AD ≅ side DC (sides of a rhombus)
∴ $\angle DAC \cong \angle DCA$ (Isosceles triangle theorem) ½
∴ $\angle DAC = \angle DCA = 35^\circ$
∴ $\angle ADC = 110^\circ$ (Remaining angle)
$\angle ADC \cong \angle ABC$ (opp. angles of a rhombus are congruent) ½
∴ $\angle ABC = 110^\circ$

(5) Equation of x-axis is '$y = 0$'. 1

(6) Equation of a line passing through '–4' on x-axis and parallel to y axis is $x = -4$ 1

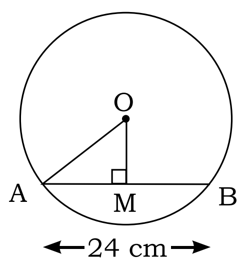
A.1. (B) Solve the following : (Any 2)

(1)

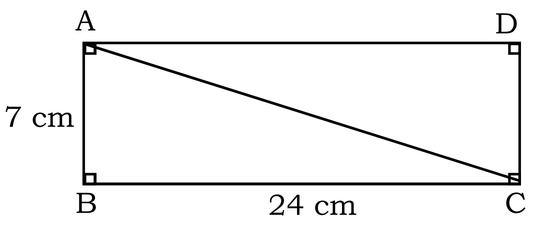


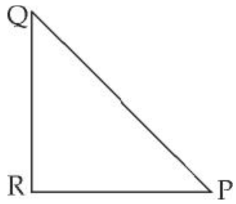
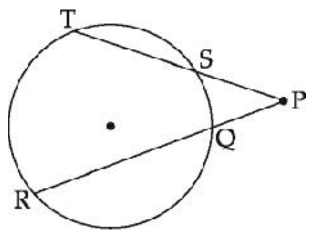
☐ WXYZ is a parallelogram ...(Given)
$\angle XYZ \cong \angle XWZ$
...(Opposite angles of a parallelogram are congruent)
But $\angle XYZ = 135^\circ$
∴ $\boxed{\angle XWZ = 135^\circ}$ ½
∴ $\angle YZW + \angle XYZ = 180^\circ$
...(Adjacent angles of a parallelogram are supplementary)
∴ $\angle YZW + 135 = 180$
∴ $\angle YZW = 180 - 135$
∴ $\boxed{\angle YZW = 45^\circ}$ ½
$l(OY) = \frac{1}{2}\, l(WY)$ ...(Diagonals of parallelogram bisect each other.) ½
∴ $5 = \frac{1}{2}\, l(WY)$
∴ $\boxed{l(WY) = 10 \text{ cm}}$ ½

(2) Diameter of the circle = 26 cm ...(Given)



Radius = $\dfrac{\text{Diameter}}{2} = \dfrac{26}{2}$ ½
∴ Radius of the circle = 13 cm
∴ PC = 13 cm

	Seg PM $\perp$ chord CD ...(Given)	
	$\therefore$ CM = $\frac{1}{2}$ CD ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord)	
	CM = $\frac{1}{2} \times 24 = 12$ cm	½
	In ΔPMC, $\angle$PMC = 90° ...(Given)	
	$\therefore$ $PC^2 = PM^2 + CM^2$ ...(Pythagoras theorem)	½
	$\therefore$ $13^2 = PM^2 + 12^2$	
	$\therefore$ $169 - 144 = PM^2$	
	$\therefore$ $PM^2 = 25$	
	$\therefore$ **PM = 5 cm** ...(Taking square roots)	½
(3)		
	Let □ABCD be a given rectangle.	½
	AB = 7 cm, BC = 24 cm	
	In ΔABC,	
	$\angle$ABC = 90° ....(Angle of a rectangle)	½
	By Pythagoras theorem,	
	$AC^2 = AB^2 + BC^2$	½
	$\therefore$ $AC^2 = 7^2 + 24^2$	
	$\therefore$ $AC^2 = 49 + 576$	
	$\therefore$ $AC^2 = 625$	
	$\therefore$ $AC = \sqrt{625}$	
	$\therefore$ AC = 25 cm	
	The length of the diagonal is 25 cm.	½
A.2.	**(A) Solve the following MCQs :**	
(1)	(C) Right angled triangle	1
(2)	(A) 7	1
(3)	(C) 5	1
(4)	(B) 13	1

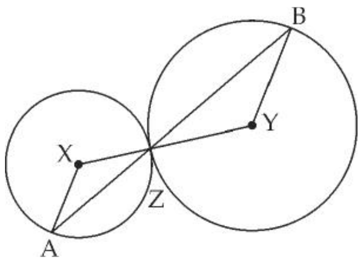
A.2.	**(B) Solve the following : (Any 2)**	
(1)	$PQ^2 = (\sqrt{8})^2 = 8$ ...(i)	½
	$PR^2 + QR^2 = (\sqrt{3})^2 + (\sqrt{5})^2$	½
	∴ $PR^2 + QR^2 = 3 + 5$	
	∴ $PR^2 + QR^2 = 8$ ...(ii)	
	∴ $PQ^2 = PR^2 + QR^2$ ...[From (i) and (ii)]	½
	Yes, ΔPQR is a right angled triangled.	
	∴ ∠R = 90° ...(Converse of Pythagoras theorem)	½
	∴ **∠R = 90°**	
		
(2)	PR = PQ + QR (P – Q – R)	
	∴ PR = 6 + 10	
	∴ PR = 16 units ...(i)	½
	Secants PST and PQR intersect each other in the exterior of the circle at point P.	
	∴ PT × PS = PR × PQ ...(Theorem of external division of chords)	
	∴ PT × 8 = 16 × 6 ...[From (i) and given]	½
	∴ $PT = \frac{16 \times 6}{8}$	
	∴ PT = 12 units ...(ii)	½
	∴ PT = PS + TS ...(P – S – T)	
	∴ 12 = 8 + TS	
	∴ TS = 12 – 8	
	∴ **TS = 4 units**	½
		
(3)	Let A(–7, 6) = (x_1, y_1) be the vertices of ΔABC	½
	B(2, –2) = (x_2, y_2)	
	C(8, 5) = (x_3, y_3)	
	Let G(x, y) be the centroid of ΔABC.	
	By centroid formula,	
	$x = \frac{x_1 + x_2 + x_3}{3}$ and $y = \frac{y_1 + y_2 + y_3}{3}$	½
	$= \frac{-7+2+8}{3}$ and $= \frac{6-2+5}{3}$	½
	$= \frac{3}{3}$ and $= \frac{9}{3}$	

$x = 1$ and $y = 3$ ∴

G(1, 3) ½

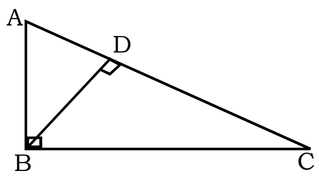
A.3. (A) Solve the following activity : (Any 2)

(1)

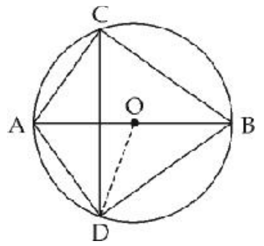


Construction :
Draw segments
XZ and YZ
Proof :
By theorem of touching circles, points X, Z, Y are collinear points
$\angle XZA \cong \angle BZY$ ...(Vertically Opposite angles)
Let $\angle XZA = \angle BZY = a$ ...(i)
seg XA ≅ seg XZ Radii of the same circle
∴ $\angle XAZ = \angle YBZ = a$ ...(ii) (Isosceles triangle theorem)
seg YB ≅ YZ Radii of the same circle
∴ $\angle BZY = \angle YBZ = a$ ...(iii) (Isosceles triangle theorem)
$m\angle XAZ = m\angle YBZ = a$ ...[From (i), (ii) and (iii)]
∴ Radius XA || radius YB alternate angles test

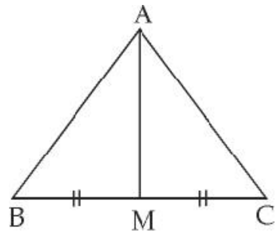
(2) (1) In ΔABC, $\angle ABC = 90°$
(2) seg BD ⊥ hypotenuse AC, A - D - C



To Prove :
$\Delta ABC \sim \Delta ADB \sim \Delta BDC$
Proof :
In ΔABC and ΔADB,
$\angle ABC \cong \angle ADB$ Each is a right angle
$\angle A \cong \angle A$ Common angle
∴ $\Delta ABC \sim \Delta ADB$ ...(i) (By AA Test of similarity)
In ΔABC and ΔBDC,

	∠ABC ≅ ∠BDC ...(Each is a right angle) ∠C ≅ ∠C ...(Common angle) ∴ ΔABC ~ ΔBDC ...(ii) By AA Test of similarity ∴ ΔABC ~ ΔADB ~ ∠BDC ...[From (i) and (ii)]	
(3)	 Proof : Draw seg OD. ∠ACB = 90° (∵ Angle inscribed in a semicircle) ∠DCB = 45° (∵ CD bisects ∠ACB) m(arc DB) = 90° ...(Inscribed angle theorem) ∠DOB = 90° ...(i) (Definition of measure of an arc) seg OA ≅ seg OB ... (ii) Radii of same circle seg OD is Perpendicular bisector of seg AB [From (i) and (ii)] ∴ seg AD ≅ seg BD Perpendicular bisector theorem	
A.3.	**(B) Solve the following : (Any 2)**	
(1)	P(k, 0) and Q(–3, –2)	½
	Slope of line PQ = $\frac{-2-0}{-3-k} = \frac{-2}{-3-k}$	½
	But slope of line PQ is given to be $\frac{2}{7}$.	½
	∴ $\frac{-2}{-3-k} = \frac{2}{7}$	
	∴ $k = 4$	½

(2) In ΔABC



$\therefore$ $BM = \frac{1}{2} BC$ ...(M is the midpoint of BC)

$\therefore$ $BM = \frac{1}{2} \times 24 = 12$ units ...(i) ½

In ΔABC, seg AM is the median ...(Given)

$\therefore$ $AB^2 + AC^2 = 2AM^2 + 2BM^2$

...(By Apollonius theorem) ½

$\therefore$ $22^2 + 34^2 = 2(AM^2 + BM^2)$

$\therefore$ $484 + 1156 = 2(AM^2 + 12^2)$

$\therefore$ $\frac{1640}{2} = AM^2 + 144$ ½

$\therefore$ $820 - 144 = AM^2$

$AM^2 = 676$

$\therefore$ **AM = 26 units** ...(Taking square roots) ½

(3) Given : ☐PQRS is a rectangle ½

To Prove : ☐PQRS is a cyclic quadrilateral

Proof : ☐PQRS is a rectangle ...(Given)

$\therefore$ $\angle P = \angle Q = \angle R = \angle S = 90°$ ...(i)

(Angles of rectangle) ½

$\angle P + \angle R = 90° + 90°$ [From (i)] ½

$\therefore$ $\angle P + \angle R = 180°$

☐PQRS is cyclic quadrilateral.

(converse of cyclic quadrilateral theorem) ½

A.4. Solve the following questions : (Any 3)

(1) $R(1, -4) = (x_1, y_1)$

$S(-2, 2) = (x_2, y_2)$

$T(-3, 4) = (x_3, y_3)$

Slope of line RS $= \frac{y_2 - y_1}{x_2 - x_1}$ ½

$= \frac{2-(-4)}{-2-1}$

$= \frac{2+4}{-3}$

$= \frac{6}{-3}$

$\therefore$ Slope of line RS = –2 ...(i) ½

Slope of line ST $= \dfrac{y_3 - y_2}{x_3 - x_2}$

$= \dfrac{4 - 2}{-3 - (-2)}$ ½

$= \dfrac{2}{-3 + 2}$

$= \dfrac{2}{-1}$

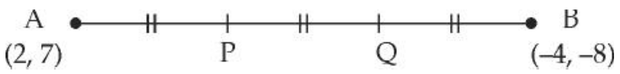
$\therefore$ Slope of line ST = –2 ...(ii) ½

$\therefore$ Slope of line RS = Slope of line ST ...[From (i) and (ii)] ½

Also, they have a common point S.

$\therefore$ **Points R, S and T are collinear points.** ½

(2)



Let point P and Q be two points which divide seg AB in three equal parts.

Point P divides seg AB in the ratio 1 : 2

By Section formula,

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$P\left(\frac{1 \times (-4) + 2 \times 2}{1+2}, \frac{1 \times (-8) + 2 \times 7}{1+2}\right)$$

$$\therefore \quad P\left(\frac{-4+4}{3}, \frac{-8+14}{3}\right)$$

$$\therefore \quad P\left(\frac{0}{3}, \frac{6}{3}\right)$$

$\therefore$ P (0, 2)

Also, PQ = QB

$\therefore$ Point Q is midpoint of seg PB.

By midpoint formula,

$$\therefore \quad Q\left(\frac{0+(-4)}{2}; \frac{2+(-8)}{2}\right)$$

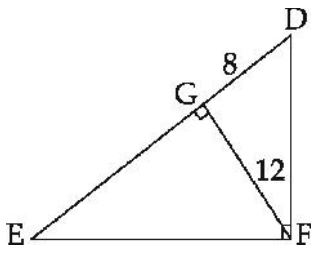
$$\therefore \quad Q\left(\frac{-4}{2}, \frac{-6}{2}\right)$$

∴ Q (–2, –3)

∴ **P(0, 2) and Q(–2, –3) are points which trisects seg AB**

(3) (i) In ΔDFE, ∠DFE = 90° ...(Given)

∴ seg FG ⊥ hypotenuse DE ...(Given)



$FG^2 = DG \times EG$ ...(Theorem of Geometric mean) ½

∴ $12^2 = 8 \times EG$

∴ $EG = \frac{12 \times 12}{8}$

∴ EG = 18 units ...(i) ½

(ii) In ΔFGD, ∠FGD = 90° ...(Given)

∴ $FD^2 = FG^2 + GD^2$ ...(By Pythagoras theorem) ½

∴ $= 12^2 + 8^2$

∴ $= 144 + 64$

∴ $FD^2 = 208$ ½

∴ $FD = \sqrt{208}$ ...(Taking square roots)

∴ $FD = \sqrt{16 \times 13}$

∴ **$FD = 4\sqrt{13}$ units**

(iii) In ΔFGE, m∠FGE = 90° ...(Given)

∴ $EF^2 = EG^2 + FG^2$ ...(By Pythagoras theorem) ½

∴ $= 18^2 + 12^2$ ...[From (i)]

∴ $= 324 + 144$

∴ $EF^2 = 468$

∴ $EF = \sqrt{468}$ (Taking square roots)

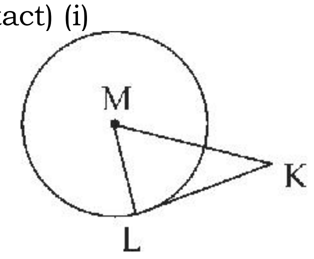
∴ $EF = \sqrt{36 \times 13}$

∴ **$EF = 6\sqrt{13}$ units** ½

(4) (i) In ΔMLK, ∠MLK = 90^0

...(Tangent and radius ⊥ at the point of contact) (i) ½

∴ $MK^2 = ML^2 + LK^2$...(Pythagoras theorem) ½



∴ $12^2 = ML^2 + (6\sqrt{3})^2$

∴ $144 = ML^2 + 108$

∴ $ML^2 = 144 - 108$

∴ $ML^2 = 36$

∴ ML = 6 units ...(ii) (Taking square roots) ½

∴ **Radius of the circle is 6 units.** ½

(ii) In ΔMLK, ∠MLK = 90° ...[From (i)] ½

ML = $\frac{1}{2}$ MK...[From (ii) and given] ½

∴ ∠K = 30° ...(Converse of 30°-60°-90° theorem) ½

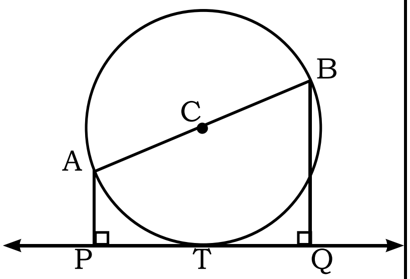
∴ ∠M = 60° ...(Sum of all angles of a triangle is 180°) ½

A.5. Solve the following questions : (Any 1)

(1) Construction : ½

Draw seg CT,

seg CP and seg CQ. ½



Proof :

seg AP ⊥ line PQ ...(i) (Given)

seg CT ⊥ line PQ ...(ii) (Tangent Theorem)

seg BQ ⊥ line PQ ...(iii) (Given)

∴ seg AP || seg CT || seg BQ ...[Perpendiculares drawn to the same line are parallel to each other from (i), (ii) and (iii)] ½

On transversals AB and PQ, $\frac{PT}{QT} = \frac{AC}{BC}$ ...(iv) ½

[Property of intercepts made by three parallel lines]

But, AC = BC ...(Radii of the same circle)

∴ $\frac{AC}{BC} = 1$ ...(v) ½

∴ $\frac{PT}{QT} = 1$ ...(vi) ... [From (iv) and (v)]

∴ PT = QT ...(vi) ½

In DPTC and DQTC,

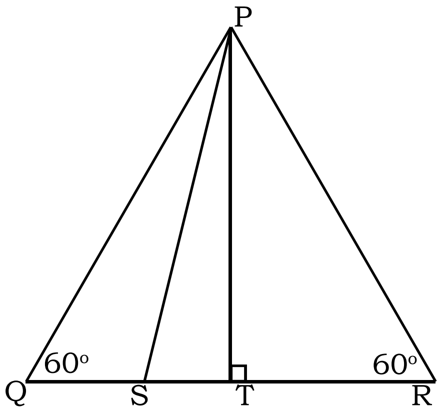
seg CT ≅ seg CT ...(Common side)

∠CTP ≅ ∠CTQ ...[Each is 90° from (ii)]

seg PT ≅ seg QT ... [From (vi)]

∴ DPTC ≅ DQTC ...(By SAS test of congruency) 1

∴ **seg CP ≅ seg CQ** ...(c.s.c.t.) ½

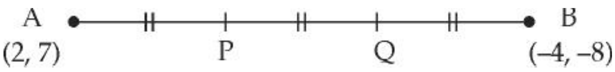
(2)		½
	$PQ = QR = PR$ ...(i) [sides of an equilateral triangle]	
	In ΔPTS, $\angle PTS = 90^\circ$ ...(Construction)	
	$\therefore \quad PS^2 = PT^2 + ST^2$ ...(ii) (By Pythagoras theorem)	
	In ΔPTQ,	
	$\angle PTQ = 90^\circ$ ...(Construction)	
	$\angle PQT = 60^\circ$ ...(angle of an equilateral triangle)	
	$\angle QPT = 30^\circ$ ...(remaining angle)	
	$\therefore \quad \Delta PTQ$ is a $30^\circ - 60^\circ - 90^\circ$ triangle	½
	By $30^\circ - 60^\circ - 90^\circ$ triangle theorem,	
	$\therefore \quad PT = \frac{\sqrt{3}}{2} PQ$ ...(iii) (side opposite to 60°)	½
	$\therefore \quad QT = \frac{1}{2} PQ$ ...(iv) (Side opposite to 30°)	½
	$ST = QT - QS$ ...(Q – S – T)	
	$\therefore \quad ST = \frac{1}{2} PQ - \frac{1}{3} QR$ ...[From (iv) and given]	½
	$\therefore \quad ST = \frac{1}{2} PQ - \frac{1}{3} PQ$ ...[From (i)]	
	$\therefore \quad ST = \frac{3PQ - 2PQ}{6}$	
	$\therefore \quad ST = \frac{1}{6} PQ$ ...(v)	½
	$\therefore \quad PS^2 = \left(\frac{\sqrt{3}}{2} PQ\right)^2 + \left(\frac{1}{6} PQ\right)^2$ ...[From (ii) (iii) and (v)]	
	$\therefore \quad PS^2 = \frac{3PQ^2}{4} + \frac{PQ^2}{36}$	½
	$\therefore \quad PS^2 = \frac{27PQ^2 + PQ^2}{36}$	

$\therefore \quad PS^2 = \dfrac{28PQ^2}{36}$

$\therefore \quad PS^2 = \dfrac{7}{9} PQ^2$

$\therefore$ **$9\ PS^2 = 7\ PQ^2$** ½

A.6. Solve the following questions : (Any 1)

(1) 

½

Let point P and Q be two points which divide seg AB in three equal parts.
Point P divides seg AB in the ratio 1 : 2
By Section formula,

$P\left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$ ½

$P\left(\dfrac{1\times(-4) + 2\times 2}{1+2}, \dfrac{1\times(-8) + 2\times 7}{1+2}\right)$ ½

$\therefore \quad P\left(\dfrac{-4+4}{3}, \dfrac{-8+14}{3}\right)$

$\therefore \quad P\left(\dfrac{0}{3}, \dfrac{6}{3}\right)$ ½

$\therefore$ P (0, 2)

Also, PQ = QB

$\therefore$ Point Q is midpoint of seg PB.

By midpoint formula,

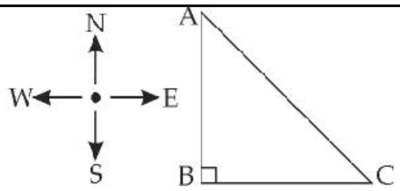
$\therefore \quad Q\left(\dfrac{0+(-4)}{2}; \dfrac{2+(-8)}{2}\right)$ ½

$\therefore \quad Q\left(\dfrac{-4}{2}, \dfrac{-6}{2}\right)$

$\therefore$ Q (–2, –3)

$\therefore$ **P(0, 2) and Q(–2, –3) are points which trisects seg AB** ½

(2)



B represents starting point of journey.

BA is the distance travelled by Prasad in North direction.

BC is the distance travelled by Pranali in east direction. 1

AC is the distance between Pranali and Prasad after two hours.

Let the speed of each one be x km/hr.

$\therefore$ Distance travelled by each one hour is $2x$ km. ½

i.e. AB = BC = $2x$ km

In ΔABC, $\angle B = 90^\circ$ ...(Line joining adjacent direction are $\perp$ to each other)

$\therefore$ $AB^2 + BC^2 = AC^2$ ...(By Pythagoras theorem) ½

$\therefore$ $(2x)^2 + (2x)^2 = (15\sqrt{2})^2$

$\therefore$ $4x^2 + 4x^2 = 225 \times 2$

$\therefore$ $8x^2 = 225 \times 2$ ½

$\therefore$ $x^2 = \frac{225 \times 2}{8}$

$\therefore$ $x^2 = \frac{225}{4}$

$\therefore$ $x = \frac{15}{2}$ ...(Taking square roots)

$\therefore$ $x = 7.5$

$\therefore$ **Speed of each one is 7.5 km / hr** ½