MT

2018 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 5

Time : 2 Hours **(Model Answer Paper)** **Max. Marks : 40**

A.1. (A) Solve the following : (Any 4) **1**

(1) Perimeter of a parallelogram = 2 (sum of lengths of adjacent sides) ½

= 2 (3 + 4) =14 cm ½

∴ Perimeter of a parallelogram is 14 cm

(2) Longest chord of a circle is Diameter ½

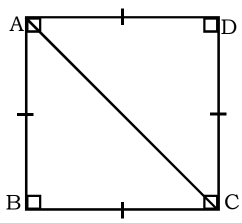
Diameter = 2 × radius

= 2 × 8

= 16

∴ Length of longest chord of a circle is 16 cm. ½

(3) Let □ABCD be a square



AB = BC = 5 cm

In ΔABC,

∠ABC = 90° (Angle of a square)

$AC^2 = AB^2 + BC^2$ [Pythagoras theorem] ½

$= 5^2 + 5^2$

$= 25 + 25$

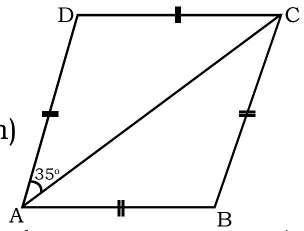
∴ $AC^2 = 50$

∴ $AC = \sqrt{50} = \sqrt{25 \times 2}$

∴ $AC = 5\sqrt{2}$

Length of the diagonal is $5\sqrt{2}$ cm ½

(4) □ABCD is a rhombus



In ΔDAC,

side AD ≅ side DC (sides of a rhombus)

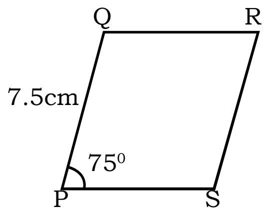
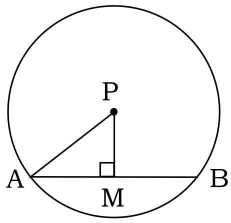
∴ ∠DAC ≅ ∠DCA (Isosceles triangle theorem) ½

∴ ∠DAC = ∠DCA = 35°

∴ ∠ADC = 110° (Remaining angle)

∠ADC ≅ ∠ABC (opp. angles of a rhombus are congruent) ½

∴ ∠ABC = 110°

(5)	Equation of x-axis is '$y = 0$'.	1
(6)	Equation of a line passing through '–4' on x-axis and parallel to y axis is $x = -4$	1
A.1.	**(B) Solve the following : (Any 2)**	
(1)	 □ PQRS is a rhombus. ...(Given) ∴ QR = PQ ...(Sides of a rhombus are equal) But, PQ = 7.5 ...(Given) ∴ **[QR = 7.5]** ∴ ∠SRQ ≅ ∠QPS ...(Opposite angles of a rhombus are congruent) But, ∠QPS = 75° ...(Given) ∴ **[∠SRQ = 75°]** □ PQRS is a parallelogram ...(Every rhombus is a parallelogram) ∴ ∠PQR + ∠SPQ = 180° ...(Adjacent angles of a parallelogram are supplementary) ∴ ∠PQR + 75 = 180 ∴ ∠PQR = 180 – 75 ∴ **[∠PQR = 105°]**	
(2)	PA = 34 cm ...(Radius of the circle) In ΔPMA, ∠PMA = 90° ...(Given)  ∴ $PA^2 = PM^2 + AM^2$ ...(Pythagoras theorem) ∴ $34^2 = 30^2 + AM^2$ ∴ $AM^2 = 34^2 - 30^2$ ∴ $AM^2 = (34 + 30)(34 - 30)$ ∴ $AM^2 = 64 \times 4$ ∴ $AM^2 = 256$ ∴ AM = 16 cm ...(Taking square roots) Seg PM ⊥ chord AB ...(Given) $AM = \frac{1}{2} \times AB$ ...(Perpendicular drawn	

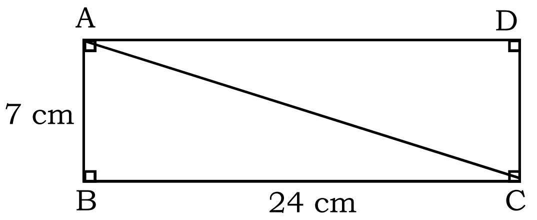
from the centre of the circle to the chord bisects the chord)

∴ $16 = \frac{1}{2} \times AB$

∴ $AB = 16 \times 2$

∴ **AB= 32 cm**

(3)



Let □ABCD be a given rectangle. ½

AB = 7 cm, BC = 24 cm

In ΔABC,

$\angle ABC = 90^\circ$ ....(Angle of a rectangle) ½

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$ ½

∴ $AC^2 = 7^2 + 24^2$

∴ $AC^2 = 49 + 576$

∴ $AC^2 = 625$

∴ $AC = \sqrt{625}$

∴ AC = 25 cm

The length of the diagonal is 25 cm. ½

A.2. (A) Solve the following MCQs :

(1) (A) 15/08/17 1

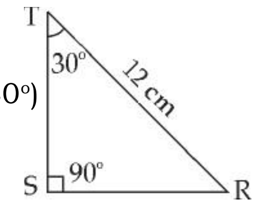
(2) (C) Three 1

(3) (C) $\frac{1}{\sqrt{3}}$ 1

(4) (B) 30 cm 1

A.2. **(B) Solve the following : (Any 2)**

(1)



In ΔPSR, $\angle S = 90^\circ$ ...(Given) ½

$\angle T = 30^\circ$ ...(Given)

$\therefore$ $\angle R = 60^\circ$ ...(Sum of all angles of a triangle is 180°)

$\therefore$ ΔPSR is $30^\circ - 60^\circ - 90^\circ$ triangle

By $30^\circ - 60^\circ - 90^\circ$ triangle theorem, ½

$RS = \frac{1}{2} \times RT$

$\therefore$ $RS = \frac{1}{2} \times 12$ ...(side opposite to 30°) ½

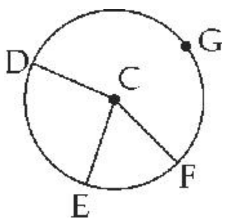
$\therefore$ **RS = 6 cm**

$ST = \frac{\sqrt{3}}{2} \times RT$ ...(side opposite to 60°)

$ST = \frac{\sqrt{3}}{2} \times 12$

$\therefore$ **ST = $6\sqrt{3}$ cm** ½

(2) ½



$\therefore$ m(arc EF) = m$\angle$ECF (Definition of measure of a minor arc)

$\therefore$ m(arc EF) = 70° ...(i) ½

m(arc DE) + m(arc EF) + m(arc DGF) = 360°

...(Measure of a circle is 360°)

$\therefore$ m(arc DE) + 70° + 200° = 360° ...[From (i) and given]

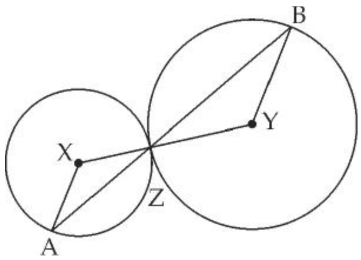
$\therefore$ m(arc DE) = $360^\circ - 270^\circ$

$\therefore$ **m(arc DE) = 90°** ½

m(arc DEF) + m(arc DE) + m(arc EF) ...(Arc addition property)

$\therefore$ m(arc DEF) = $90^\circ + 70^\circ$ ...[From (i) and given]

$\therefore$ **m(arc DEF) = 160°** ½

(3)	P(–5, 7) = (x_1, y_1)	½
	Q(–1, 3) = (x_2, y_2)	
	By distance formula,	½
	d(P, Q) = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	
	= $\sqrt{\left[-1-(-5)\right]^2 + (3-7)^2}$	
	= $\sqrt{(-1+5)^2 + (-4)^2}$	½
	= $\sqrt{(4)^2 + 16}$	
	= $\sqrt{16+16}$	½
	= $\sqrt{32}$	
	= $\sqrt{16 \times 2}$	
	∴ **d(P, Q) = $4\sqrt{2}$ units**	
A.3. (1)	**(A) Solve the following activity : (Any 2)**	
		
	Construction : Draw segments	
	XZ and YZ	
	Proof :	
	By theorem of touching circles, points X, Z, Y are collinear points	
	∠XZA ≅ ∠BZY ...(Vertically Opposite angles)	
	Let ∠XZA = ∠BZY = a ...(i)	
	seg XA ≅ seg XZ Radii of the same circle	
	∴ ∠XAZ = ∠YBZ = a ...(ii) (Isosceles triangle theorem)	
	seg YB ≅ YZ Radii of the same circle	

∴ ∠BZY = ∠YBZ = a ...(iii) (Isosceles triangle theorem)

m∠XAZ = m∠YBZ = a ...[From (i), (ii) and (iii)]

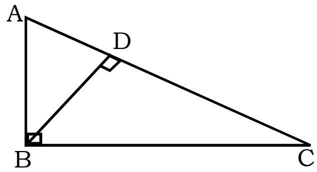
∴ Radius XA ∥ radius YB alternate angles test

(2) (1) In ΔABC, ∠ABC = 90°

(2) seg BD ⊥ hypotenuse AC, A - D - C

To Prove :

ΔABC ~ ΔADB ~ ΔBDC



Proof :

In ΔABC and ΔADB,

∠ABC ≅ ∠ADB Each is a right angle

∠A ≅ ∠A Common angle

∴ ΔABC ~ ΔADB ...(i) (By AA Test of similarity)

In ΔABC and ΔBDC,

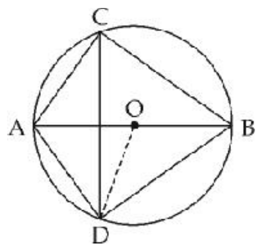
∠ABC ≅ ∠BDC ...(Each is a right angle)

∠C ≅ ∠C ...(Common angle)

∴ ΔABC ~ ΔBDC ...(ii) By AA Test of similarity

∴ ΔABC ~ ΔADB ~ ∠BDC ...[From (i) and (ii)]

(3)



Proof : Draw seg OD.

∠ACB = 90° (∵ Angle inscribed in a semicircle)

∠DCB = 45° (∵ CD bisects ∠ACB)

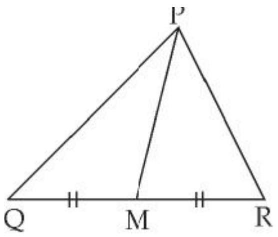
m(arc DB) = 90° ...(Inscribed angle theorem)

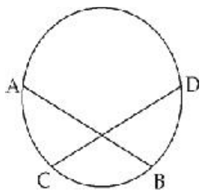
∠DOB = 90° ...(i) (Definition of measure of an arc)

seg OA ≅ seg OB ... (ii) Radii of same circle

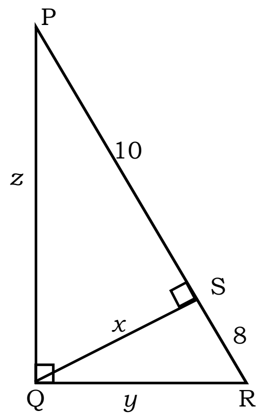
seg OD is Perpendicular bisector of seg AB [From (i) and (ii)]

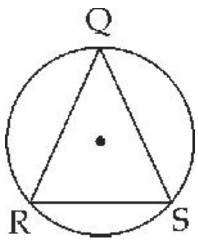
∴ seg AD ≅ seg BD Perpendicular bisector theorem

A.3.	**(B) Solve the following : (Any 2)**	
(1)	P(–12, –3), Q(4, k) ...(Given)	½
	Slope of PQ = $\frac{1}{2}$ ...(Given)	
	Slope of PQ = $\frac{k-(-3)}{4-(-12)}$ ...(Given)	½
	$\therefore \quad \frac{1}{2} = \frac{k+3}{4+12}$	
	$\therefore \quad \frac{16}{2} = k + 3$	½
	$\therefore \quad k + 3 = 8$	
	$\therefore \quad k = 8 - 3$	
	$\therefore$ **k = 5**	½
(2)		½
	In ΔPQR, seg PM is the median ...(Given)	
	$\therefore \quad PQ^2 + PR^2 = 2PM^2 + 2QM^2$ ...(Appollonius theorem)	
	$\therefore \quad 40^2 + 42^2 = 2\,(29)^2 + 2(QM)^2$	½
	$\therefore \quad (40)^2 + (42)^2 = 2\,(29^2 + QM^2)$	
	$\therefore \quad 1600 + 1764 = 2\,(841 + QM^2)$	
	$\therefore \quad \frac{3364}{2} = 841 + QM^2$	
	$\therefore \quad 1682 - 841 = QM^2$	½
	$\therefore \quad QM^2 = 841$	
	$\therefore \quad QM = 29$ ...(Taking square roots)	
	$QR = 2QM$ ...(M is midpoint of seg QR)	
	$\therefore \quad QR = 2 \times 29$	
	$\therefore$ **QR = 58 units**	½

(3)	 Proof : chord AB ≅ chord CD ...(Gven) arc ACB ≅ arc DBC [In a circle, congruent chords have their corresponding minor arcs are congruent] m (arc ACB) = m (arc DBC) m (arc AC) + m (arc CB) = m (arc CB) + m (arc BD) ...(Arc addition property) m (arc AC) = m (arc BD) **arc AC ≅ arc BD**	1 ½ ½
A.4.	**Solve the following questions : (Any 3)**	
(1)	P(–2, 3) = (x_1, y_1) Q(1, 2) = (x_2, y_2) R(4, 1) = (x_3, y_3) Slope of line PQ = $\frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{2-3}{1-(-2)}$ $= \frac{-1}{1+2}$ ∴ Slope of line PQ $= \frac{-1}{3}$ ...(i) Slope of line QR $= \frac{y_3 - y_2}{x_3 - x_2}$ $= \frac{1-2}{4-1}$ ∴ Slope of line QR $= \frac{-1}{3}$ ...(ii) ∴ Slope of line PQ = slope of line QR ...[From (i) and (ii)] Line PQ and line QR have equal slopes and have a common point Q. ∴ **Points P, Q and R are collinear**	½ ½ ½ ½ ½ ½

(2)	A(1, 2), B(1, 6) and C(1 + $2\sqrt{3}$, 4) be the vetices of triangle Using distance formula,	½
	d(A, B) = $\sqrt{(1-1)^2+(2-6)^2}$	
	= $\sqrt{0^2+(-4)^2}$	
	= $\sqrt{0+16}$	
	= $\sqrt{16}$	
	∴ d(A, B) = 4 units ...(i)	½
	d(B, C) = $\sqrt{(1+2\sqrt{3}-1)^2+(4-6)^2}$	
	= $\sqrt{\left(2\sqrt{3}\right)^2+(-2)^2}$	½
	= $\sqrt{12+4}$	
	∴ d(B, C) = $\sqrt{16}$	
	d(B, C) = 4 units ...(ii)	½
	d(A, C) = $\sqrt{(1+2\sqrt{3}-1)^2+(4-2)^2}$	
	= $\sqrt{(2\sqrt{3})^2+(2)^2}$	
	= $\sqrt{12+4}$	½
	= $\sqrt{16}$	
	∴ d(A, C) = 4 units ...(iii)	
	∴ AB = BC = AC ...[From (i), (ii) and (iii)]	
	∴ **ΔABC is an equilateral triangle** ...(By Definition)	½
(3)	In ΔPQR, ∠PQR = 90°, seg QS ⊥ seg PR	½
	QS = $\sqrt{PS \times SR}$ (theorem of geometric mean)	½
	= $\sqrt{10 \times 8}$	
	= $\sqrt{5\times2\times8}$	
	= $\sqrt{5\times16}$	
	= $4\sqrt{5}$	
	∴ x = $4\sqrt{5}$	½
	In ΔQSR, by Pythagoras theorem	
	QR^2 = $QS^2 + SR^2$	

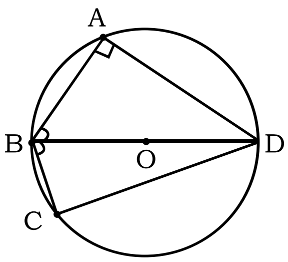


	$= \left(4\sqrt{5}\right)^2 + 8^2$ $= 16 \times 5 + 64$ $= 80 + 64$ $= 144$ $\therefore$ QR $= 12$	½
	In ΔPSQ, by Pythagoras theorem $PQ^2 = QS^2 + PS^2$ $= \left(4\sqrt{5}\right)^2 + 10^2$ $= 16 \times 5 + 100$ $= 80 + 100$ $= 180$ $= 36 \times 5$ $\therefore$ PQ $= 6\sqrt{5}$	
	Hence $x = 4\sqrt{5}$, $y = 12$, $z = 6\sqrt{5}$	½
(4)		½
	Proof : ΔQRS is an equilateral triangle. ...(Given) $\therefore$ chord QR ≅ chord RS ≅ chord QS ...(Sides of equilateral Δ are equal) arc RS ≅ arc QS ≅ arc QR...(i) (In circle, congruent chords have corresponding minor arcs are congruent] Let m (arc RS) = m (arc QS) = m (arc QR) = x ...(ii) [From (i) and supposition]	½
	$\therefore$ m (arc RS) + m (arc QS) + m (arc QR) = 360° ...(Measure of a circle is 360°)	½
	$\therefore$ $x + x + x = 360°$ $\therefore$ $3x = 360°$ $\therefore$ $x = 120°$	½
	$\therefore$ m (arc RS) = m (arc QS) + m (arc QR) = 120° ...(iii)	

∴ m (arc QRS) = m (arc QR) + m (arc RS) ...(Arc addition property) ½
∴ m (arc QRS) = $120^\circ + 120^\circ$ ...[From (iii)]
∴ **m(arc QRS) = 240°** ½

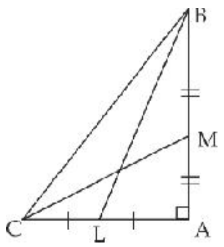
A.5. Solve the following questions : (Any 1)

(1) Prove that any 3 points on a circle cannot be collinear.
Given : O is the centre of circle.
A, B, C are any 3 points on it.
To Prove : Points A, B, C are not collinear.
Construction : Draw seg AB, BC.
Also, draw diameter BD, seg AD, seg CD



Proof : In ΔBAD, ∠BAD = 90°
[Angle inscribed is a semicircle is a right angle]
∴ ∠ABD < 90° ... (i)
Similarly, by drawing seg CD, we can prove
∠CBD < 90° ... (ii)
∠ABD + ∠CBD < 90 + 90 [Adding (i) and (ii)]
∴ ∠ABD + ∠CBD < 180°
Prove that any 3 points on a circle cannot be collinear.
Given : O is the centre of circle.
A, B, C are any 3 points on it.
To Prove: Points A, B, C are not collinear.
Construction : Draw seg AB, BC.
Also, draw diameter BD, seg AD, seg CD.
Proof :
∴ ∠ABD + ∠CBD < 180°
∴ ∠ABD + ∠CBD ≠ 180°
∴ ∠ABD and ∠CBD do not form a linear pair
∴ Ray BA and Ray BC are not opposite rays

(2) ½



To Prove :

$4(BL^2 + CM^2) = 5BC^2$ ...(Given) ½

Proof :

In ΔBAC, $\angle BAC = 90^\circ$ ...(Given)

$\therefore \quad BC^2 = AB^2 + AC^2$ ...(i) (By Pythagoras theorem) ½

In ΔBAL, $\angle BAC = 90^\circ$ ...(Given)

$\therefore \quad BL^2 = AB^2 + AL^2$ ...(ii) (By Pythagoras theorem) ½

In ΔCAM, $\angle CAM = 90^\circ$ ...(Given)

$\therefore \quad CM^2 = AC^2 + AM^2$ ...(iii) (By Pythagoras theorem)

Adding (ii) and (iii),

$BL^2 + CM^2 = AB^2 + AL^2 + AC^2 + AM^2$ ½

$\therefore \quad BL^2 + CM^2 = AB^2 + AC^2 + AL^2 + AM^2$

$\therefore \quad BL^2 + CM^2 = BC^2 + AL^2 + AM^2$ [From (i)] ½

$\therefore \quad BL^2 + CM^2 = BC^2 + \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}AB\right)^2$

[$\because$ L and M are the midpoint of sides AC and AB respectively]

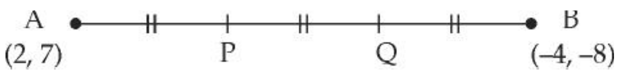
$\therefore \quad BL^2 + CM^2 = BC^2 + \frac{AC^2}{4} + \frac{AB^2}{4}$ ½

$\therefore \quad 4\,(BL^2 + CM^2) = 4BC^2 + AC^2 + AB^2$ (Multiplying throughout by 4)

$\therefore \quad 4\,(BL^2 + CM^2) = 4BC^2 + BC^2$ ...[From (i)]

$\therefore \quad \mathbf{4\,(BL^2 + CM^2) = 5BC^2}$

A.6. Solve the following questions : (Any 1)

(1) 

½

Let point P and Q be two points which divide seg AB in three equal parts.

Point P divides seg AB in the ratio 1 : 2

By Section formula,

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$ ½

$$P\left(\frac{1\times(-4) + 2\times 2}{1+2}, \frac{1\times(-8) + 2\times 7}{1+2}\right)$$ ½

$\therefore \quad P\left(\frac{-4+4}{3}, \frac{-8+14}{3}\right)$

$\therefore \quad P\left(\frac{0}{3}, \frac{6}{3}\right)$ ½

$\therefore \quad$ P (0, 2)

Also, PQ = QB

$\therefore \quad$ Point Q is midpoint of seg PB.

By midpoint formula,

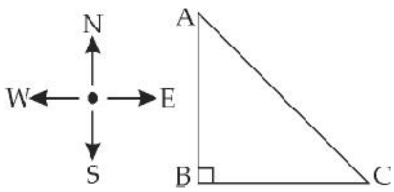
$\therefore \quad Q\left(\frac{0+(-4)}{2}; \frac{2+(-8)}{2}\right)$ ½

$\therefore \quad Q\left(\frac{-4}{2}, \frac{-6}{2}\right)$

$\therefore \quad$ Q (–2, –3)

$\therefore \quad$ **P(0, 2) and Q(–2, –3) are points which trisects seg AB** ½

(2)



B represents starting point of journey.

BA is the distance travelled by Prasad in North direction.

BC is the distance travelled by Pranali in east direction. 1

AC is the distance between Pranali and Prasad after two hours.

Let the speed of each one be x km/hr.

$\therefore \quad$ Distance travelled by each one hour is $2x$ km. ½

i.e. AB = BC = $2x$ km

In ΔABC, $\angle B = 90^\circ$ ...(Line joining adjacent direction are $\perp$ to each other)

$\therefore \quad AB^2 + BC^2 = AC^2$ ...(By Pythagoras theorem) ½

$\therefore \quad (2x)^2 + (2x)^2 = (15\sqrt{2})^2$

$\therefore \quad 4x^2 + 4x^2 = 225 \times 2$

$\therefore \quad 8x^2 = 225 \times 2$ ½

	$\therefore \quad x^2 = \dfrac{225 \times 2}{8}$	
	$\therefore \quad x^2 = \dfrac{225}{4}$	
	$\therefore \quad x = \dfrac{15}{2}$ ...(Taking square roots)	
	$\therefore \quad x = 7.5$	
	$\therefore$ **Speed of each one is 7.5 km / hr**	½