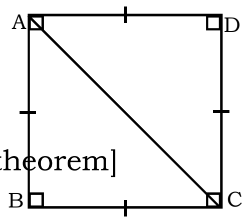
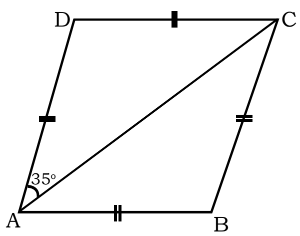
MT

2018 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 4

Time : 2 Hours **(Model Answer Paper)** **Max. Marks : 40**

A.1.	**(A) Solve the following : (Any 4)** **1**	
(1)	Perimeter of a parallelogram = 2 (sum of lengths of adjacent sides)	½
	= 2 (3 + 4) =14 cm	½
	∴ Perimeter of a parallelogram is 14 cm	
(2)	Longest chord of a circle is Diameter	½
	Diameter = 2 × radius = 2 × 8 = 16	
	∴ Length of longest chord of a circle is 16 cm.	½
(3)	Let □ABCD be a square AB = BC = 5 cm In ΔABC, ∠ABC = 90° (Angle of a square)  	
	$AC^2 = AB^2 + BC^2$ [Pythagoras theorem]	½
	$= 5^2 + 5^2$ $= 25 + 25$ ∴ $AC^2 = 50$ ∴ $AC = \sqrt{50} = \sqrt{25 \times 2}$ ∴ $AC = 5\sqrt{2}$	
	Length of the diagonal is $5\sqrt{2}$ cm	½
(4)	 	

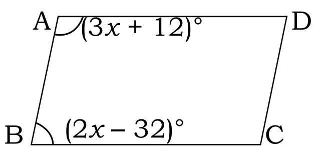
□ABCD is a rhombus
In ΔDAC,
side AD ≅ side DC (sides of a rhombus)
$\therefore$ $\angle DAC \cong \angle DCA$ (Isosceles triangle theorem) ½
$\therefore$ $\angle DAC = \angle DCA = 35^\circ$
$\therefore$ $\angle ADC = 110^\circ$ (Remaining angle)
$\angle ADC \cong \angle ABC$ (opp. angles of a rhombus are congruent) ½
$\therefore$ $\angle ABC = 110^\circ$

(5) Equation of x-axis is '$y = 0$'. 1

(6) Equation of a line passing through '–4' on x-axis and parallel to y axis is $x = -4$ 1

A.1. (B) Solve the following : (Any 2)

(1) ½

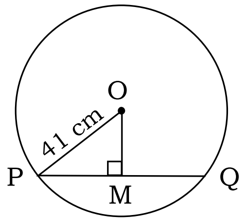


□ ABCD is a parallelogram ...(Given) ½
$\therefore$ $\angle A + \angle B = 180^\circ$ ...(Adjacent angles of a parallelogram are supplementary)
$\therefore$ $(3x + 12) + (2x - 32) = 180$
$\therefore$ $5x - 20 = 180$
$\therefore$ $5x = 180 + 20$
$\therefore$ $5x = 200$
$\therefore$ $\boxed{x = 40}$ ½

$\angle A = (3x + 12)$	$\angle B = (2x - 32)$
$= 3(40) + 12$	$= 2(40) - 32$
$= 120 + 12$	$= 80 - 32$
$\therefore$ $\angle A = 132^\circ$	$\therefore$ $\angle B = 48^\circ$

$\therefore$ $\angle C = \angle A$ and $\angle D = \angle B$ ...(Opposite angles of a parallelogram are congruent.)
$\therefore$ $\boxed{\angle C = 132^\circ \text{ and } \angle D = 48^\circ}$ ½

(2) ½



OP = 41 cm ...(Radius of the circle)

$\therefore$ Seg OM $\perp$ chord PQ ...(Given) ½

$\therefore$ PM = $\frac{1}{2}$ PQ ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

$\therefore$ PM = $\frac{1}{2} \times 80 = 40$ cm

In ΔOMP, $\angle$OMP = 90° ...(Given)

$\therefore$ $OP^2 = OM^2 + PM^2$ ...(Pythagoras theorem)

$\therefore$ $41^2 = OM^2 + 40^2$

$\therefore$ $OM^2 = 41^2 - 40^2$

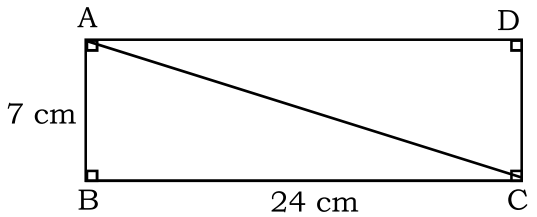
$\therefore$ $OM^2 = (41 + 40)(41 - 40)$ ½

$\therefore$ $OM^2 = 81 \times 1$

$\therefore$ $OM^2 = 81$

$\therefore$ **OM = 9 cm** ...(Taking square roots) ½

(3)



Let □ABCD be a given rectangle. ½

AB = 7 cm, BC = 24 cm

In ΔABC,

$\angle$ABC = 90° ....(Angle of a rectangle) ½

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$ ½

$\therefore$ $AC^2 = 7^2 + 24^2$

$\therefore$ $AC^2 = 49 + 576$

$\therefore$ $AC^2 = 625$

$\therefore$ AC = $\sqrt{625}$

$\therefore$ AC = 25 cm

The length of the diagonal is 25 cm. ½

A.2. (A) Solve the following MCQs :

(1) (D) 40 cm — 1

(2) (B) 72 — 1

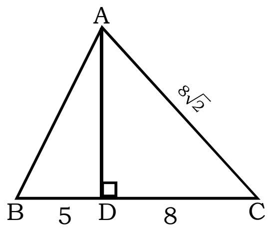
(3) (D) (1, –3) — 1

(4) (C) 6 cm — 1

A.2. (B) Solve the following : (Any 2)

(1) In ΔADC — ½

$\angle ADC = 90^\circ$, $\angle C = 45^\circ$, $\therefore \angle DAC = 45^\circ$



$AD = DC = \frac{1}{\sqrt{2}} \times 8\sqrt{2}$ by $45^\circ - 45^\circ - 90^\circ$ theorem, — ½

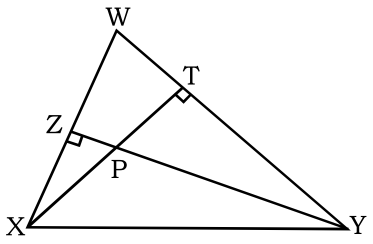
$DC = 8 \therefore$ $\boxed{AD = 8}$ — ½

$BC = BD + DC$

$= 5 + 8$

$\boxed{BC = 13}$ — ½

(2) — ½



Proof :

$\angle XTW = 90^\circ$ ...(i) (Given) — ½

$\angle YZW = 90^\circ$ ...(ii) (Given)

$\angle XTW + \angle YZW = 90^\circ + 90^\circ$ ...(Adding (i) and (ii))

$\therefore$ $\angle PTW + \angle PZW = 180^\circ$(X – P – T, Y – P – Z)

□WZPT is a cyclic quadrilateral ...(Convers of cyclic quadrilateral theorem)

$\angle XZY = \angle XTY = 90^\circ$ ...(Given)

$\therefore$ $\angle XZY = \angle XTY$ — ½

$\therefore$ seg XY subtends congruent angle at points Z and T which are on the same side of line XY.

∴ **Point X, Z, T and Y are concyclic points.** ½

(3) C(5, –2) = (x_1, y_1) ½

D(7, 3) = (x_2, y_2)

Slope of line CD = $\dfrac{y_2 - y_1}{x_2 - x_1}$

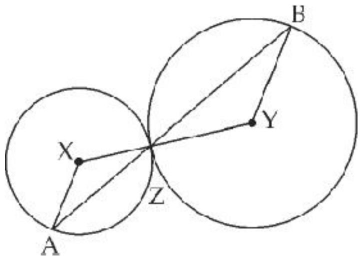
$= \dfrac{3-(-2)}{7-5}$ ½

$= \dfrac{3+2}{2} = \dfrac{5}{2}$ ½

∴ **Slope of line CD = $\dfrac{5}{2}$** ½

A.3. (A) Solve the following activity : (Any 2)

(1)



Construction :

Draw segments

XZ and YZ

Proof :

By theorem of touching circles, points X, Z, Y are collinear points

∠XZA ≅ ∠BZY ...(Vertically Opposite angles)

Let ∠XZA = ∠BZY = a ...(i)

seg XA ≅ seg XZ Radii of the same circle

∴ ∠XAZ = ∠YBZ = a ...(ii) (Isosceles triangle theorem)

seg YB ≅ YZ Radii of the same circle

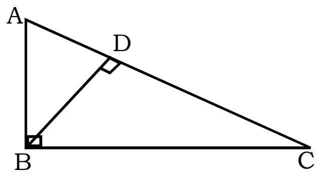
∴ ∠BZY = ∠YBZ = a ...(iii) (Isosceles triangle theorem)

m∠XAZ = m∠YBZ = a ...[From (i), (ii) and (iii)]

∴ Radius XA || radius YB alternate angles test

(2) (1) In ΔABC, $\angle ABC = 90^\circ$

(2) seg BD $\perp$ hypotenuse AC, A - D - C



To Prove :

$\Delta ABC \sim \Delta ADB \sim \Delta BDC$

Proof :

In ΔABC and ΔADB,

$\angle ABC \cong \angle ADB$ Each is a right angle

$\angle A \cong \angle A$ Common angle

$\therefore$ ΔABC ~ ΔADB ...(i) (By AA Test of similarity)

In ΔABC and ΔBDC,

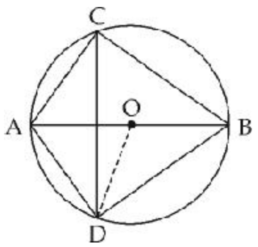
$\angle ABC$ $\cong$ $\angle BDC$ ...(Each is a right angle)

$\angle C$ $\cong$ $\angle C$ ...(Common angle)

$\therefore$ $\Delta ABC \sim \Delta BDC$ ...(ii) By AA Test of similarity

$\therefore$ ΔABC ~ ΔADB ~ $\angle BDC$ ...[From (i) and (ii)]

(3)



Proof : Draw seg OD.

$\angle ACB$ = 90° ($\because$ Angle inscribed in a semicircle)

$\angle DCB$ = 45° ($\because$ CD bisects $\angle ACB$)

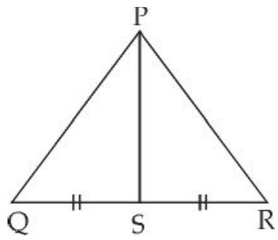
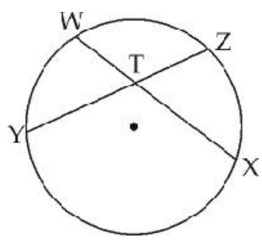
m(arc DB) = 90° ...(Inscribed angle theorem)

$\angle DOB$ = 90° ...(i) (Definition of measure of an arc)

seg OA $\cong$ seg OB ... (ii) Radii of same circle

seg OD is Perpendicular bisector of seg AB [From (i) and (ii)]

$\therefore$ seg AD $\cong$ seg BD Perpendicular bisector theorem

A.3.	**(B) Solve the following : (Any 2)**	
(1)	P(–2, 0) = (x, y)	½
	A(2, –3) = (x_1, y_1)	
	B(x_2, y_2)	
	P is the centre of the circle ...(Given)	½
	Point P is the midpoint of diameter AB.	
	By midpoint formula,	
	$x = \frac{x_1 + x_2}{2}$ and $y = \frac{y_1 + y_2}{2}$	
	$-2 = \frac{2 + x_2}{2}$ and $0 = \frac{-3 + y_2}{2}$	
	∴ $-2 \times 2 = 2 + x_2$ and $0 \times 2 = -3 + y_2$	
	∴ $-4 - 2 = x_2$ and $0 + 3 = y_2$	
	∴ $x_2 = -6$ and $y_2 = 3$	
	∴ **B(–6, 3)**	1
(2)	In ΔPQR, PS is the median ...(By Definition)	½
	∴ $PQ^2 + PR^2 = 2PS^2 + 2QS^2$ ...(Apollonius theorem)	
	∴ $11^2 + 17^2 = 2 \times (13^2 + QS^2)$	
	∴ $121 + 289 = 2 (169 + QS^2)$ 	½
	∴ $\frac{410}{2} = 169 + QS^2$	
	∴ $205 - 169 = QS^2$	
	∴ $QS^2 = 36$	½
	∴ QS = 6 units ...(Taking square roots)	
	QR = 2 QS ...(∵ S is the midpoint of seg QR, given)	
	∴ QR = 2 × 6	
	∴ **QR=12 units**	½
(3)	YZ = YT + TZ ...(Y – T – Z) 	½
	∴ 26 = 8 + TZ	
	∴ TZ = 26 – 8	
	∴ TZ = 18	½
	Let WT = ‘a’	
	WX = WT + TX [W–T–X]	
	∴ 25 = a + TX	

$\therefore$ TX = (25 – a)

Chords XW and YZ intersect each other in the interior of the circle at point T. ½

WT × TX = TZ × YT

$\therefore$ $a(25 - a) = 18 \times 8$

$\therefore$ $25a - a^2 = 144$

$\therefore$ $a^2 - 25a + 144 = 0$

$\therefore$ $a^2 - 16a - 9a + 144 = 0$

$\therefore$ $a(a - 16) - 9(a - 16) = 0$

$\therefore$ $(a - 16)(a - 9) = 0$

$\therefore$ $a - 16 = 0$ or $a - 9 = 0$

$\therefore$ $a = 16$ or $a = 9$

$\therefore$ **WT = 16 units** or **WT = 9 units** ½

A.4. **Solve the following questions : (Any 3)**

(1) $P(2, -5) = (x_1, y_1)$

$Q(1, -3) = (x_2, y_2)$

$R(-2, 3) = (x_3, y_3)$

Slope of line PQ $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-3-(-5)}{1-2}$

$= \dfrac{-3+5}{-1}$

$= \dfrac{2}{-1}$

$\therefore$ Slope of line PQ $= -2$ ...(i) ½

Slope of line QR $= \dfrac{y_3 - y_2}{x_3 - x_2}$

$= \dfrac{3-(-3)}{-2-1}$ ½

$= \dfrac{\text{[illegible]} +}{-\text{[illegible]}}$

$= \dfrac{6}{-3}$

$\therefore$ Slope of line QR $= -2$ ...(ii) ½

$\therefore$ Slope of line PQ = Slope of line QR ...[From (i) and (ii)]

Also, both lines have a common point Q.

∴ **Points P, Q and R are collinear points.** ½

(2) Line PQ || Line RS ...(Given) ½

∴ Slope of line PQ = Slope of line RS $\frac{6-4}{3-2} = \frac{k-1}{5-3}$

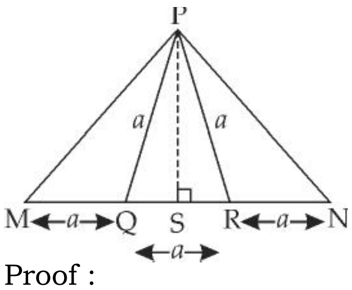
∴ $\frac{2}{1} = \frac{k-1}{2}$ ½

∴ $2 \times 2 = k - 1$ ½

∴ $4 + 1 = k$

∴ $\boldsymbol{k = 5}$ ½

(3) ½



Proof :

MQ = QR = RN = a ...(Given)

Point Q is the midpoint of seg MR ...(i) ½

∴ In ΔPMR, seg PQ is a median ...[From (i), Definition]

∴ $PM^2 + PR^2 = 2PQ^2 + 2QM^2$ ...(Apollonius theorem) ½

∴ $PM^2 + a^2 = 2a^2 + 2a^2$

∴ $PM^2 = 4a^2 - a^2$ ½

∴ $PM^2 = 3a^2$

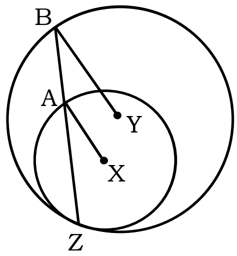
∴ $\mathbf{PM} = \sqrt{3}\,\boldsymbol{a}$ ...(Taking square roots) ½

Similarly we can prove, PN = $\sqrt{3}a$

∴ PM = PN = $\sqrt{3}a$ ½

(4) Construction : ½

Draw seg ZY.



Proof :

Y-X-Z (Theorem of touching circle) ½

In ΔXAZ, seg XA $\cong$ seg XZ (Radii of the same circle)

$\therefore$ $\angle XAZ \cong \angle XZA$ (Isosceles triangle theorem) ½

$\therefore$ $\angle XAZ \cong \angle YZB$ ...(i) [Y-X-Z, Z-A-B]

In ΔYBZ, seg YB $\cong$ seg YZ (Radii of same circle) ½

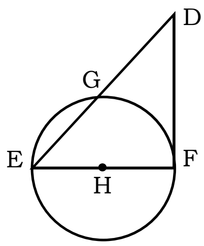
$\therefore$ $\angle YBZ \cong \angle YZB$ ...(ii) (Isosceles triangle theorem)

$\therefore$ $\angle XAZ \cong \angle YBZ$ ...[From (i) and (ii)] ½

$\therefore$ seg AX || seg BY ...(Corresponding angles test) ½

A.5. Solve the following questions : (Any 1)

(1) ½



To prove: DE × GE - $4r^2$ ½

Construction : Draw seg GF

Proof : $\angle EGF = 90°$ ...(i)

(Diameter subtends a right angle at any point on the circle) ½

In ΔEFD,

$\angle EFD = 90°$ ...(Tangent and radius are perpendicular at the point of contact) ½

seg FG $\perp$ hypotenuse ED ...[From (i)]

$\therefore$ $\Delta EFD \sim \Delta EGF \sim \Delta FGD$ ...(ii)

(similarity in right angled triangles) ½

$\therefore$ $\Delta EFD \sim \Delta EGF$ ...[From (ii)] ½

$\therefore$ $\frac{EF}{GE} = \frac{DE}{EF}$ ...(c.s.c.t.) ½

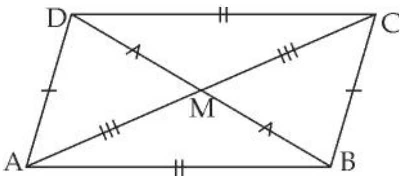
$\therefore$ $DE \times GE = EF^2$

$\therefore$ $DE \times GE = (2r)^2$ ...(diameter is twice the radius)

$\therefore$ $\mathbf{DE \times GE = 4r^2}$ ½

(2) Given :

In □ABCD, is a parallelogram
Diagonals AC and BD intersect each other at point M. ½



To Prove :

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$ (5 mark) ½

Proof :

□ABCD is a parallelogram ...(Given)

$AM = CM = \frac{1}{2} AC$ ...(i)

$BM = DM = \frac{1}{2} BD$ ...(ii)

[Diagonals of a parallelogram bisect each other] ½

$\therefore$ In ΔABC, seg BM is a median ...[From (i)] ½

$\therefore$ $AB^2 + BC^2 = 2BM^2 + 2AM^2$ ...(iii) (By Apollonius theorem)

$\therefore$ In ΔADC, seg DM is a median ...[From (i)] ½

$\therefore$ $CD^2 + AD^2 = 2DM^2 + 2AM^2$ ...(iv) (By Apollonius theorem)

Adding (iii) and (iv)

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 2BM^2 + 2AM^2 + 2DM^2 + 2AM^2$

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 2BM^2 + 2DM^2 + 4AM^2$

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 2BM^2 + 2BM^2 + 4AM^2$ ...[From (ii)]

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 4BM^2 + 4AM^2$

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 4 [BM^2 + AM^2]$ ½

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 4 \left[\left(\frac{1}{2}BD\right)^2 + \left(\frac{1}{2}AC\right)^2\right]$ ...[From (i) and (ii)]

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 4 \left[\frac{1}{4}BD^2 + \frac{1}{4}AC^2\right]$

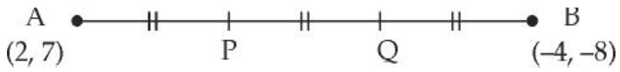
$\therefore$ $= 4 \times \frac{1}{4} [BD^2 + AC^2]$

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = BD^2 + AC^2$

$\therefore$ $\mathbf{BD^2 + AC^2 = AB^2 + BC^2 + CD^2 + AD^2}$ ½

A.6. Solve the following questions : (Any 1)

(1)



½

Let point P and Q be two points which divide seg AB in three equal parts.

Point P divides seg AB in the ratio 1 : 2

By Section formula,

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$ ½

$$P\left(\frac{1\times(-4)+2\times 2}{1+2}, \frac{1\times(-8)+2\times 7}{1+2}\right)$$ ½

$$\therefore \quad P\left(\frac{-4+4}{3}, \frac{-8+14}{3}\right)$$

$$\therefore \quad P\left(\frac{0}{3}, \frac{6}{3}\right)$$ ½

$\therefore$ P (0, 2)

Also, PQ = QB

$\therefore$ Point Q is midpoint of seg PB.

By midpoint formula,

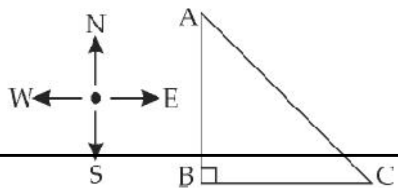
$$\therefore \quad Q\left(\frac{0+(-4)}{2}; \frac{2+(-8)}{2}\right)$$ ½

$$\therefore \quad Q\left(\frac{-4}{2}, \frac{-6}{2}\right)$$

$\therefore$ Q (–2, –3)

$\therefore$ **P(0, 2) and Q(–2, –3) are points which trisects seg AB** ½

(2)



	B represents starting point of journey. BA is the distance travelled by Prasad in North direction. BC is the distance travelled by Pranali in east direction. AC is the distance between Pranali and Prasad after two hours. Let the speed of each one be x km/hr. $\therefore$ Distance travelled by each one hour is $2x$ km. i.e. AB = BC = $2x$ km In ΔABC, $\angle B = 90^\circ$ ...(Line joining adjacent direction are $\perp$ to each other) $\therefore$ $AB^2 + BC^2 = AC^2$ ...(By Pythagoras theorem) $\therefore$ $(2x)^2 + (2x)^2 = \left(15\sqrt{2}\right)^2$ $\therefore$ $4x^2 + 4x^2 = 225 \times 2$ $\therefore$ $8x^2 = 225 \times 2$ $\therefore$ $x^2 = \frac{225 \times 2}{8}$ $\therefore$ $x^2 = \frac{225}{4}$ $\therefore$ $x = \frac{15}{2}$ ...(Taking square roots) $\therefore$ $x = 7.5$ $\therefore$ **Speed of each one is 7.5 km / hr**	1 ½ ½ ½ ½