MT

2018 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 3

Time : 2 Hours **(Model Answer Paper)** **Max. Marks : 40**

A.1. (A) Solve the following : (Any 4) **1**

(1) Perimeter of a parallelogram = 2 (sum of lengths of adjacent sides) ½

= 2 (3 + 4) =14 cm ½

∴ Perimeter of a parallelogram is 14 cm

(2) Longest chord of a circle is Diameter ½

Diameter = 2 × radius

= 2 × 8

= 16

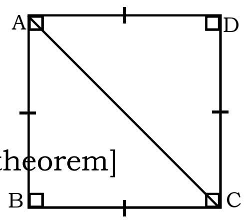
∴ Length of longest chord of a circle is 16 cm. ½

(3) Let □ABCD be a square

AB = BC = 5 cm

In ΔABC,

$\angle ABC = 90^\circ$ (Angle of a square)



$AC^2 = AB^2 + BC^2$ [Pythagoras theorem] ½

$= 5^2 + 5^2$

$= 25 + 25$

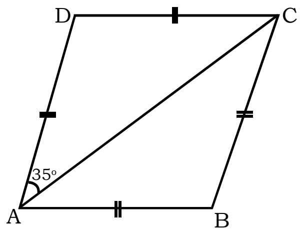
∴ $AC^2 = 50$

∴ $AC = \sqrt{50} = \sqrt{25 \times 2}$

∴ $AC = 5\sqrt{2}$

Length of the diagonal is $5\sqrt{2}$ cm ½

(4) $\square$ABCD is a rhombus



In ΔDAC,

side AD $\cong$ side DC (sides of a rhombus)

$\therefore$ $\angle$DAC $\cong$ $\angle$DCA (Isosceles triangle theorem) ½

$\therefore$ $\angle$DAC = $\angle$DCA = 35°

$\therefore$ $\angle$ADC = 110° (Remaining angle)

$\angle$ADC $\cong$ $\angle$ABC (opp. angles of a rhombus are congruent) ½

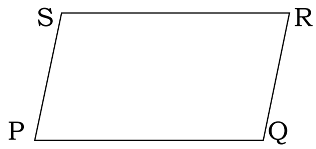
$\therefore$ $\angle$ABC = 110°

(5) Equation of x-axis is '$y = 0$'. 1

(6) Equation of a line passing through '–4' on x-axis and parallel to y axis is $x = -4$ 1

A.1. (B) Solve the following : (Any 2)

(1) $\square$ PQRS is a parallelogram. ½



$\angle$P : $\angle$Q = 1 : 2

Let the common multiple be x.

$\therefore$ $\angle$P = x and $\angle$Q = $2x$

$\angle$P + $\angle$Q = 180° ½

(Adjacent angles of a parallelogram are supplementary)

$\therefore$ $x + 2x = 180$

$\therefore$ $3x = 180$

$\therefore$ $x = \dfrac{180}{3} = 60$ ½

$\therefore$ $\angle P = x = 60°$

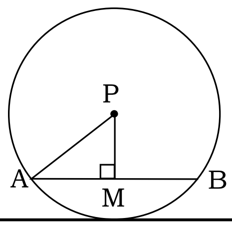
$\angle Q = 2x = 2 \times 60 = 120°$

$\therefore$ $\angle$R = $\angle$P and $\angle$S = $\angle$Q

...(Opposite angles of a parallelogram are congruent)

$\therefore$ $\angle$R = 60° and $\angle$S = 120° ½

(2) PA = 34 cm...(Radius of the circle)



In ΔPMA, $\angle$PMA = 90° ...(Given)

$\therefore$ $PA^2 = PM^2 + AM^2$ ...(Pythagoras theorem)

$\therefore$ $34^2 = 30^2 + AM^2$

$\therefore$ $AM^2 = 34^2 - 30^2$

$\therefore$ $AM^2 = (34 + 30)(34 - 30)$

$\therefore$ $AM^2 = 64 \times 4$

$\therefore$ $AM^2 = 256$

$\therefore$ AM = 16 cm ...(Taking square roots)

Seg PM $\perp$ chord AB ...(Given)

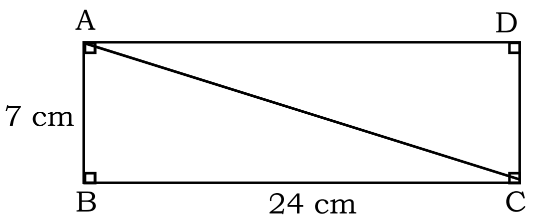
$AM = \frac{1}{2} \times AB$ ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord)

$\therefore$ $16 = \frac{1}{2} \times AB$

$\therefore$ $AB = 16 \times 2$

$\therefore$ **AB= 32 cm**

(3)



Let □ABCD be a given rectangle. ½

AB = 7 cm, BC = 24 cm

In ΔABC,

$\angle ABC = 90^\circ$ ....(Angle of a rectangle) ½

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$ ½

$\therefore$ $AC^2 = 7^2 + 24^2$

$\therefore$ $AC^2 = 49 + 576$

$\therefore$ $AC^2 = 625$

$\therefore$ $AC = \sqrt{625}$

$\therefore$ AC = 25 cm

The length of the diagonal is 25 cm. ½

A.2. (A) Solve the following MCQs :

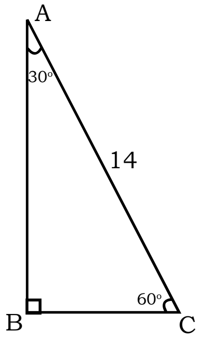
(1)	(C) Right angled triangle	1
(2)	(B) 12 cm	1
(3)	(D) (2,0)	1
(4)	(A) 30°	1

A.2. (B) Solve the following : (Any 2)

(1) In ΔABC ½

$\angle B = 90^{o}$, $\angle A = 30^{o}$, $\therefore \angle C = 60^{o}$

By $30^{o} - 60^{o} - 90^{o}$ theorem, ½



$BC = \frac{1}{2} \times AC$

$BP = \frac{1}{2} \times 14$

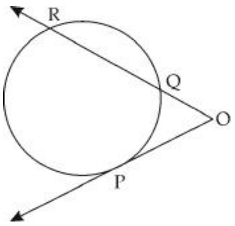
BC = 7 ½

$AB = \frac{\sqrt{3}}{2} \times AC$

$AB = \frac{\sqrt{3}}{2} \times 14$

BC = 7 $\sqrt{3}$ ½

(2) (i) $\angle POR = \frac{1}{2}$ [marc PR – marc PQ] ½



$\therefore$ $36^{\circ} = \frac{1}{2}$ [140° – marc PQ]

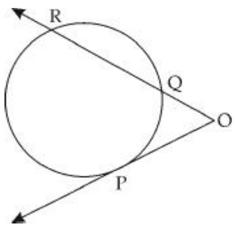
$\therefore$ $36 \times 2 = 140^{\circ}$ – marc PQ

$\therefore$ m(arc PQ) = $140^{\circ} - 72^{\circ}$

$\therefore$ **m arc PQ = 68°** ½

(ii) $OP^2 = OQ \times OR$ ...(Tangent secant segment theorem)

$\therefore$ $7.2 \times 7.2 = OQ \times 16.2$



$\therefore$ $OQ = \frac{7.2 \times 7.2}{16.2}$

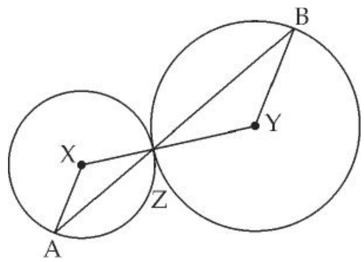
$\therefore$ OQ = 3.2 ½

OQ + QR = OR ...(O – Q – R)

$\therefore$ 3.2 + QR = 16.2

$\therefore$ QR = 16.2 – 3.2

$\therefore$ QR = 13 units ½

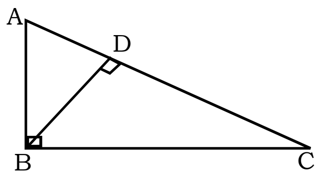
(3)	P(0, 6) = (x_1, y_1) and Q(12, 20) = (x_2, y_2) Let M (x, y) be the midpoint of seg PQ. By midpoint formula, $x = \frac{x_1 + x_2}{2}$ and $y = \frac{y_1 + y_2}{2}$ $x = \frac{0+12}{2}$ and $y = \frac{6+20}{2}$ $x = \frac{12}{2}$ and $y = \frac{26}{2}$ $x = 6$ and $y = 13$ ∴ **M(6, 13) is the midpoint of segment joining P (0, 6) and Q (12, 20)**	½ ½ ½ ½
A.3.	**(A) Solve the following activity : (Any 2)**	
(1)	 Construction : Draw segments XZ and YZ Proof : By theorem of touching circles, points X, Z, Y are collinear points ∠XZA ≅ ∠BZY ...(Vertically Opposite angles) Let ∠XZA = ∠BZY = a ...(i) seg XA ≅ seg XZ Radii of the same circle ∴ ∠XAZ = ∠YBZ = a ...(ii) (Isosceles triangle theorem) seg YB ≅ YZ Radii of the same circle ∴ ∠BZY = ∠YBZ = a ...(iii) (Isosceles triangle theorem)	

m∠XAZ = m∠YBZ = a ...[From (i), (ii) and (iii)]

∴ Radius XA || radius YB | alternate angles test |

(2) (1) In ΔABC, ∠ABC = 90°
(2) seg BD ⊥ hypotenuse AC, A - D - C

To Prove :
ΔABC ~ ΔADB ~ ΔBDC



Proof :

In ΔABC and ΔADB,

∠ABC ≅ ∠ADB | Each is a right angle |

∠A ≅ ∠A | Common angle |

∴ | ΔABC | ~ | ΔADB | ...(i) (By AA Test of similarity)

In ΔABC and ΔBDC,

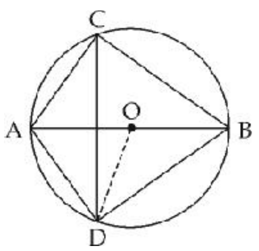
| ∠ABC | ≅ | ∠BDC | ...(Each is a right angle)

| ∠C | ≅ | ∠C | ...(Common angle)

∴ ΔABC ~ ΔBDC ...(ii) | By AA Test of similarity |

∴ ΔABC ~ | ΔADB | ~ | ∠BDC | ...[From (i) and (ii)]

(3)



Proof : Draw seg OD.

∠ACB = | 90° | (∵ Angle inscribed in a semicircle)

∠DCB = | 45° | (∵ CD bisects ∠ACB)

m(arc DB) = | 90° | ...(Inscribed angle theorem)

∠DOB = | 90° | ...(i) (Definition of measure of an arc)

seg OA ≅ seg OB ... (ii) | Radii of same circle |

seg OD is | Perpendicular bisector | of seg AB [From (i) and (ii)]

∴ seg AD ≅ seg BD | Perpendicular bisector theorem |

A.3. (B) Solve the following : (Any 2)

(1) Let P(x, 0) be a point on X axis which is equidistant from A(–3, 4) and B(1, –4). ½

$\therefore$ d(P, A) = d(P, B)

By distance formula,

$\sqrt{[x-(-3)]^2+(0-4)^2} = \sqrt{(x-1)^2+[0-(-4)]^2}$ ½

$\therefore \quad \sqrt{(x+3)^2+(-4)^2} = \sqrt{(x-1)^2+(4)^2}$

Squaring both the sides we get,

$(x + 3)^2 + 16 = (x - 1)^2 + 16$

$\therefore \quad x^2 + 6x + 9 = x^2 - 2x + 1$ ½

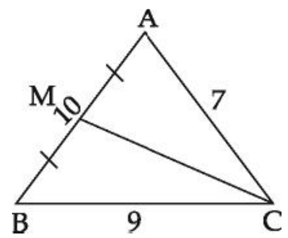
$\therefore \quad x^2 + 6x - x^2 + 2x = 1 - 9$

$\therefore \quad 8x = -8$

$\therefore \quad x = -1$

$\therefore$ **P(–1, 0) is the required point.** ½

(2) $\therefore \quad BM = \frac{1}{2} \times AB$ (M is the midpoint of seg AB) ½



$\therefore \quad BM = \frac{1}{2} \times 10$

$\therefore \quad BM = 5$ units ...(i)

In ΔABC, CM is the median ...(Given) ½

$\therefore \quad AC^2 + BC^2 = 2CM^2 + 2BM^2$ ...(Apollonius theorem)

$\therefore \quad 7^2 + 9^2 = 2(CM^2 + 5^2)$

$\therefore \quad 49 + 81 = 2(CM^2 + 25)$

$\therefore \quad \frac{130}{2} = CM^2 + 25$

$\therefore \quad CM^2 = 65 - 25$

$\therefore \quad CM^2 = 40$ ½

$\therefore \quad CM = \sqrt{40}$ ...(Taking square roots)

$\therefore \quad CM = \sqrt{4 \times 10}$

$\therefore$ $\mathbf{CM = 2\sqrt{10}}$ **units** ½

(3) □MRPN is a cyclic quadrilateral...(Given) ½

$\therefore$ $\angle R + \angle N = 180°$...(Cyclic quadrilateral theorem)

$\therefore$ $5x - 13 + 4x + 4 = 180$ ½

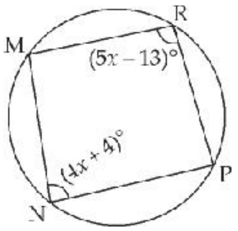
$\therefore$ $9x - 9 = 180$

$\therefore$ $9x = 180 + 9$

$\therefore$ $9x = 189$

$\therefore$ $x = \frac{189}{9}$ ½

$\therefore$ $x = 21$



$\therefore$

$m\angle R = 5x - 13 = 5 \times 21 - 13 = 105 - 13 = 92°$
$m\angle N = 4x + 4 = 4 \times 21 + 4 = 84 + 4 = 88°$

½

A.4. Solve the following questions : (Any 3)

(1) $A(1, -3) = (x_1, y_1)$ ½

$B(2, -5) = (x_2, y_2)$

$C(-4, 7) = (x_3, y_3)$ ½

Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-5-(-3)}{2-1}$

$= \frac{-5+3}{1}$ ½

$= -2$

$\therefore$ Slope of line AB $= -2$ ...(i) ½

Slope of line BC $= \frac{y_3 - y_2}{x_3 - x_2}$

$= \frac{7-(-5)}{-4-2}$ ½

$= \frac{7+5}{-6}$

$\therefore$ Slope of line BC $= \frac{12}{-6} = -2$ ...(ii)

∴ Slope of line AB = Slope of line BC ...[From (i) and (ii)]

Line AB and line BC have equal slopes and have a common point B.

∴ **Points A, B and C are collinear.** ½

(2) Suppose, point P(11,15) divides segment AB in the ratio $m : n$

by section formula, ½

$x = \dfrac{mx_2 + nx_1}{m+n}$

∴ $11 = \dfrac{9m+15n}{m+n}$ ½

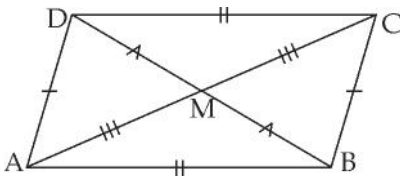
∴ $11m + 11n = 9m + 15n$ ½

∴ $2m = 4n$

∴ $\dfrac{m}{n} = \dfrac{4}{2} = \dfrac{2}{1}$ ½

∴ The required ratio is 2 : 1. ½

(3) ½



☐ABCD is a parallelogram ...(Given) ½

∴ BM = $\frac{1}{2}$ BD ...(i)

∴ AM = $\frac{1}{2}$ AC

} [Diagonals of a parallelogram bisect each other] ...(ii)

∴ AM = $\frac{1}{2} \times 14 = 7$cm ½

In ΔABC, seg BM is the a median ...[From (ii) and Definition]

∴ $AB^2 + BC^2 = 2AM^2 + 2BM^2$ ...(Apollonius theorem) ½

$30 = 2(7^2 + BM^2)$

∴ $\dfrac{130}{2} = 49 + BM^2$

∴ $65 - 49 = BM^2$ ½

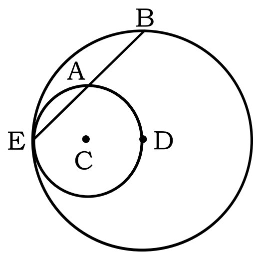
∴ $BM^2 = 16$ ½

∴ BM = 4 cm ...(Taking square roots)

∴ $\frac{1}{2}$ BD = 4cm ...[From (i)]

∴ **BD = 8 cm** ½

(4) Construction : Draw seg DE and seg DA ½



Proof : E – C – D...(Theorem on touching circles)

Seg DE is the diameter. (Definition) ½

∠DAE = 90° ...(i) 1

(Diameter subtends a right angle at any point of circle other than its end points) ½

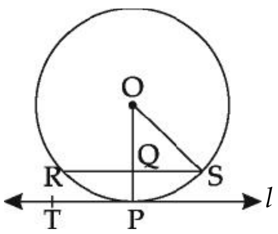
Consider circle with centre D.

∴ seg DA ⊥ chord BE ...[From (i)] ½

∴ **seg EA ≅ seg AB** ...(Perpendicular drawn from the centre to the chord bisects the chord) 1

A.5. Solve the following questions : (Any 1)

(1) ½



Take a point T on line l as shown in the figure.

∠OPT = 90° ... (i) (Tangent Theorem)

chord RS || line l ...(Given)

∴ ∠OPT ≅ ∠OQR ... (ii) (Corresponding angles theorem) ½

∴ ∠OQR = 90° ... (iii) [From (i) and (ii)]

∴ seg OQ ⊥ chord RS ... [From (iii)] ½

$\therefore$ $QR = \frac{1}{2} RS$ ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

$\therefore$ $QR = \frac{1}{2} \times 12$

$\therefore$ QR = 6 units ½

Let the radius of the circle be x units.

$\therefore$ OR = OP = x units ...(Radii of the same circle)

$\therefore$ $OQ = \frac{1}{2} OP$ ($\because$ Q is midpoint of seg OP)

$\therefore$ $OQ = \frac{1}{2} \times x$

$\therefore$ $OQ = \frac{x}{2}$ ½

In ΔOQR,

$\angle OQR = 90^\circ$ ... [From (iii)]

$\therefore$ $OR^2 = OQ^2 + QR^2$ ...(By Pythagoras theorem)

$\therefore$ $x^2 = \left(\frac{x}{2}\right)^2 + (6)^2$

$\therefore$ $x^2 = \frac{x^2}{4} + 36$

$\therefore$ $4x^2 = x^2 + 144$ ...(Multiplying throughout by 4)

$\therefore$ $4x^2 - x^2 = 144$

$\therefore$ $3x^2 = 144$ ½

$\therefore$ $x^2 = \frac{144}{3}$

$\therefore$ $x^2 = 48$

$\therefore$ $x = \sqrt{48}$ ...(taking square roots)

$\therefore$ $x = \sqrt{16 \times 3}$

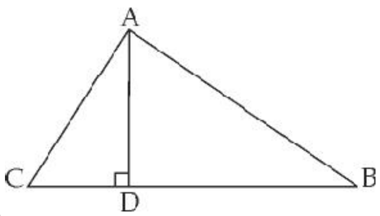
$\therefore$ $x = 4\sqrt{3}$

$\therefore$ OR = OP = $4\sqrt{3}$ units

$\therefore$ **Radius of the circle is $4\sqrt{3}$ units.** ½

(2) To Prove : ½

$2AB^2 = 2AC^2 + BC^2$



Proof :

$DB = 3CD$ ...(i) (Given) ½

In ΔADB, $\angle ADB = 90^\circ$ (Given)

$\therefore \quad AB^2 = AD^2 + DB^2$ (By Pythagoras theorem)

$\therefore \quad AB^2 = AD^2 + (3CD)^2$ [From (i)] ½

$\therefore \quad AB^2 = AD^2 + 9CD^2$ ...(ii)

In ΔADC, $\angle ADC = 90^\circ$ ...(Given) ½

$\therefore \quad AC^2 = AD^2 + CD^2$ (By Pythagoras theorem)

$\therefore \quad AD^2 = AC^2 - CD^2$ ...(iii)

$AB^2 = AC^2 - CD^2 + 9CD^2$ [From (ii) and (iii)]

$\therefore \quad AB^2 = AC^2 + 8CD^2$ ...(iv) ½

But $BC = CD + DB$ ...[C - D - B]

$\therefore \quad BC = CD + 3CD$ ...[From (i)]

$\therefore \quad BC = 4CD$

$\therefore \quad CD = \dfrac{BC}{4}$ ...(v) ½

$\therefore \quad AB^2 = AC^2 + 8\left(\dfrac{BC}{4}\right)^2$ ...[From (iv) and (v)]

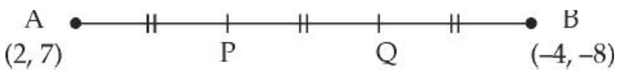
$\therefore \quad AB^2 = AC^2 + 8 \times \dfrac{BC^2}{16}$ ½

$\therefore \quad AB^2 = AC^2 + \dfrac{BC^2}{2}$ ...[From (iv) and (v)]

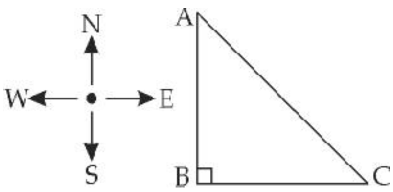
$\therefore$ $\boxed{\mathbf{2AB^2 = 2AC^2 + BC^2}}$ (Multiplying throughout by 2) ½

A.6. Solve the following questions : (Any 1)

(1) ½



Let point P and Q be two points which divide seg AB in three equal parts.

	Point P divides seg AB in the ratio 1 : 2	
	By Section formula,	
	P $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$	½
	P $\left(\frac{1\times(-4)+2\times 2}{1+2}, \frac{1\times(-8)+2\times 7}{1+2}\right)$	½
	∴ P $\left(\frac{-4+4}{3}, \frac{-8+14}{3}\right)$	
	∴ P $\left(\frac{0}{3}, \frac{6}{3}\right)$	½
	∴ P (0, 2)	
	Also, PQ = QB	
	∴ Point Q is midpoint of seg PB.	
	By midpoint formula,	
	∴ Q $\left(\frac{0+(-4)}{2}; \frac{2+(-8)}{2}\right)$	½
	∴ Q $\left(\frac{-4}{2}, \frac{-6}{2}\right)$	
	∴ Q (–2, –3)	
	∴ **P(0, 2) and Q(–2, –3) are points which trisects seg AB**	½
(2)		
	B represents starting point of journey. BA is the distance travelled by Prasad in North direction. BC is the distance travelled by Pranali in east direction. AC is the distance between Pranali and Prasad after two hours. Let the speed of each one be x km/hr.	1

	∴ Distance travelled by each one hour is $2x$ km.	½
	i.e. AB = BC = $2x$ km	
	In ΔABC, $\angle B = 90^\circ$ ...(Line joining adjacent direction are $\perp$ to each other)	
	∴ $AB^2 + BC^2 = AC^2$ ...(By Pythagoras theorem)	½
	∴ $(2x)^2 + (2x)^2 = (15\sqrt{2})^2$	
	∴ $4x^2 + 4x^2 = 225 \times 2$	
	∴ $8x^2 = 225 \times 2$	½
	∴ $x^2 = \frac{225 \times 2}{8}$	
	∴ $x^2 = \frac{225}{4}$	
	∴ $x = \frac{15}{2}$ ...(Taking square roots)	
	∴ $x = 7.5$	
	∴ **Speed of each one is 7.5 km / hr**	½