MT

2018 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 2

Time : 2 Hours **(Model Answer Paper)** **Max. Marks : 40**

A.1. (A) Solve the following : (Any 4) **1**

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(1) Perimeter of a parallelogram = 2 (sum of lengths of adjacent sides) ½

= 2 (3 + 4) =14 cm ½

∴ Perimeter of a parallelogram is 14 cm

(2) Longest chord of a circle is Diameter ½

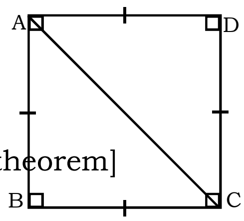
Diameter = 2 × radius

= 2 × 8

= 16

∴ Length of longest chord of a circle is 16 cm. ½

(3) Let □ABCD be a square



AB = BC = 5 cm

In ΔABC,

$\angle ABC = 90^\circ$ (Angle of a square)

$AC^2 = AB^2 + BC^2$ [Pythagoras theorem] ½

$= 5^2 + 5^2$

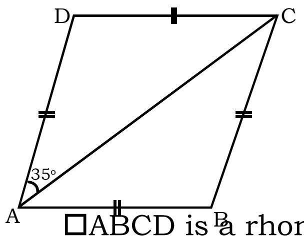
$= 25 + 25$

∴ $AC^2 = 50$

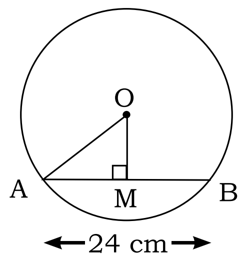
∴ $AC = \sqrt{50} = \sqrt{25 \times 2}$

∴ $AC = 5\sqrt{2}$

Length of the diagonal is $5\sqrt{2}$ cm ½

(4)	 □ABCD is a rhombus In ΔDAC, side AD ≅ side DC (sides of a rhombus) ∴ $\angle DAC \cong \angle DCA$ (Isosceles triangle theorem) ∴ $\angle DAC = \angle DCA = 35^\circ$ ∴ $\angle ADC = 110^\circ$ (Remaining angle) $\angle ADC \cong \angle ABC$ (opp. angles of a rhombus are congruent) ∴ $\angle ABC = 110^\circ$	 ½ ½
(5)	Equation of x-axis is '$y = 0$'.	1
(6)	Equation of a line passing through '–4' on x-axis and parallel to y axis is $x = -4$	1
A.1.	**(B) Solve the following : (Any 2)**	
(1)	□ABCD is a parallelogram ... (Given) $\angle A + \angle B = 180^\circ$ [Adjacent angles of a parallelogram are supplementary] ∴ $x + 3x + 20 = 180$ ∴ $4x = 180 - 20$ ∴ $4x = 160$ ∴ $x = 40$ ∴ $\angle A = 40^\circ$ $\angle A \cong \angle C$ [opposite angles of a parallelogram are congruent] ∴ $\angle C = 40^\circ$ $\angle B = 3x + 20$ $= 3(40) + 20$ ∴ $\angle B = 140^\circ$ $\angle B \cong \angle D$ [opposite angles of a parallelogram are congruent] ∴ $\angle D = 140^\circ$	½ ½ ½ ½

(2) Diameter of the circle = 26 cm ...(Given)



Radius = $\frac{\text{Diameter}}{2} = \frac{26}{2}$ ½

$\therefore$ Radius of the circle = 13 cm

$\therefore$ PC = 13 cm

Seg PM $\perp$ chord CD ...(Given)

$\therefore$ CM = $\frac{1}{2}$ CD (Perpendicular drawn from the centre of the circle to the chord bisects the chord) ½

CM = $\frac{1}{2} \times 24 = 12$ cm

In ΔPMC, $\angle$PMC = 90° (Given)

$\therefore$ $PC^2 = PM^2 + CM^2$ (Pythagoras theorem)

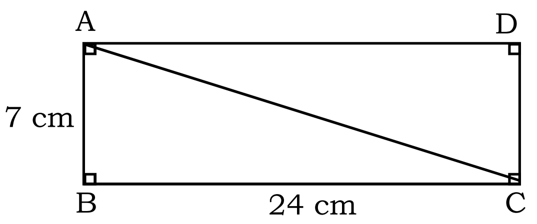
$\therefore$ $13^2 = PM^2 + 12^2$

$\therefore$ $169 - 144 = PM^2$

$\therefore$ $PM^2 = 25$

$\therefore$ **PM = 5 cm** (Taking square roots)

(3)



Let □ABCD be a given rectangle. ½

AB = 7 cm, BC = 24 cm

In ΔABC,

$\angle$ABC = 90° ....(Angle of a rectangle) ½

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\therefore$ $AC^2 = 7^2 + 24^2$

$\therefore$ $AC^2 = 49 + 576$

$\therefore$ $AC^2 = 625$

$\therefore$ AC = $\sqrt{625}$

$\therefore$ AC = 25 cm 1

The length of the diagonal is 25 cm. ½

A.2. (A) Solve the following MCQs :

(1) (B) 30 cm — 1

(2) (A) Equilateral triangle — 1

(3) $\frac{1}{\sqrt{3}}$ — 1

(4) 13 — 1

A.2. (B) Solve the following : (Any 2)

(1) In ΔABC, seg AP is the median on side BC — ½

$\therefore$ $BP = \frac{1}{2} \times BC$

$\therefore$ $BP = \frac{1}{2} \times 18$ — ½

$\therefore$ BP = 9 units ...(i)

In ΔABC, seg AP is the median ...(Given)

$\therefore$ $AB^2 + AC^2 = 2BP^2 + 2AP^2$ ...(Apollonius theorem) — ½

$\therefore$ $260 = 2(9^2 + AP^2)$ ...[From (i)]

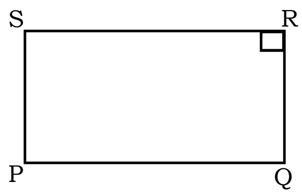
$\therefore$ $\frac{260}{2} = 81 + AP^2$

$\therefore$ $AP^2 = 130 - 81$

$\therefore$ $AP^2 = 49$

$\therefore$ **AP = 7 units** ...(Taking square roots) — ½

(2) Given : □PQRS is a rectangle — ½



To Prove : □PQRS is a cyclic quadrilateral

Proof : □PQRS is a rectangle ...(Given) — ½

$\therefore$ $\angle P = \angle Q = \angle R = \angle S = 90°$ ...(i) (Angles of rectangle)

$\angle P + \angle R = 90° + 90°$ [From (i)] — ½

$\therefore$ $\angle P + \angle R = 180°$

□PQRS is cyclic quadrilateral. (converse of cyclic quadrilateral theorem) — ½

(3) R(1, –1) = (x1, y1) ½

S(–2, k) = (x2, y2)

Slope of line RS = $\frac{y_2 - y_1}{x_2 - x_1}$ ½

$-2 = \frac{k-(-1)}{-2-1}$ ½

∴ $-2 = \frac{k+1}{-3}$

∴ (–2) × (–3) = k + 1 ½

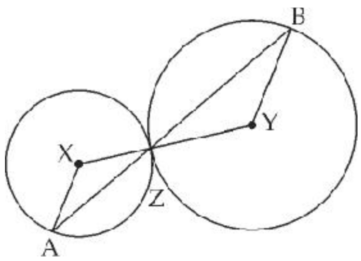
∴ 6 = k + 1

∴ k = 6 – 1

∴ **k = 5** ½

A.3. (A) Solve the following activity : (Any 2)

(1)



Construction :
Draw segments
XZ and YZ
Proof :
By theorem of touching circles, points X, Z, Y are collinear points
∠XZA ≅ ∠BZY ...(Vertically Opposite angles)
Let ∠XZA = ∠BZY = a ...(i)

seg XA ≅ seg XZ Radii of the same circle

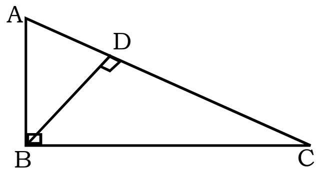
∴ ∠XAZ = ∠YBZ = a ...(ii) (Isosceles triangle theorem)

seg YB ≅ YZ Radii of the same circle

∴ ∠BZY = ∠YBZ = a ...(iii) (Isosceles triangle theorem)

m∠XAZ = m∠YBZ = a ...[From (i), (ii) and (iii)]

	∴ Radius XA ∥ radius YB alternate angles test
(2)	(1) In ΔABC, ∠ABC = 90° (2) seg BD ⊥ hypotenuse AC, A - D - C To Prove : ΔABC ~ ΔADB ~ ΔBDC



Proof :

In ΔABC and ΔADB,

∠ABC ≅ ∠ADB   Each is a right angle

∠A ≅ ∠A   Common angle

∴ ΔABC ~ ΔADB   ...(i) (By AA Test of similarity)

In ΔABC and ΔBDC,

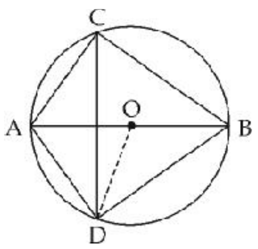
∠ABC ≅ ∠BDC   ...(Each is a right angle)

∠C ≅ ∠C   ...(Common angle)

∴ ΔABC ~ ΔBDC   ...(ii) By AA Test of similarity

∴ ΔABC ~ ΔADB ~ ∠BDC   ...[From (i) and (ii)]

(3)



Proof : Draw seg OD.

∠ACB = 90°   (∵ Angle inscribed in a semicircle)

∠DCB = 45°   (∵ CD bisects ∠ACB)

m(arc DB) = 90°   ...(Inscribed angle theorem)

∠DOB = 90°   ...(i) (Definition of measure of an arc)

seg OA ≅ seg OB   ... (ii) Radii of same circle

seg OD is Perpendicular bisector of seg AB [From (i) and (ii)]

∴ seg AD ≅ seg BD   Perpendicular bisector theorem

A.3. (B) Solve the following : (Any 2)

(1) Let P(x, 0) be a point on X axis which is equidistant from A(–3, 4) and B(1, –4). ½

$\therefore$ d(P, A) = d(P, B)

By distance formula,

$$\sqrt{[x-(-3)]^2+(0-4)^2} = \sqrt{(x-1)^2+[0-(-4)]^2}$$ ½

$$\therefore \quad \sqrt{(x+3)^2+(-4)^2} = \sqrt{(x-1)^2+(4)^2}$$

Squaring both the sides we get,

$(x + 3)^2 + 16 = (x - 1)^2 + 16$

$\therefore \quad x^2 + 6x + 9 = x^2 - 2x + 1$

$\therefore \quad x^2 + 6x - x^2 + 2x = 1 - 9$ ½

$\therefore \quad 8x = -8$

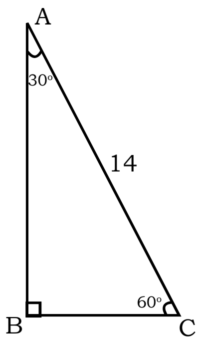
$\therefore \quad x = -1$

$\therefore$ **P(–1, 0) is the required point.** ½

(2) In ΔABC ½

$\angle B = 90^\circ$, $\angle A = 30^\circ$, $\therefore \angle C = 60^\circ$

By $30^\circ - 60^\circ - 90^\circ$ theorem,



$$BC = \frac{1}{2} \times AC$$

$$BP = \frac{1}{2} \times 14$$

BC = 7 ½

$$AB = \frac{\sqrt{3}}{2} \times AC$$

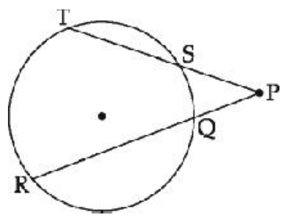
$$AB = \frac{\sqrt{3}}{2} \times 14$$ ½

BC = 7 $\sqrt{3}$ ½

(3) PR = PQ + QR (P – Q – R) ½

$\therefore$ PR = 6 + 10

$\therefore$ PR = 16 units ...(i)



Secants PST and PQR ½

	intersect each other in the exterior of the circle at point P.	
	∴ PT × PS = PR × PQ ...(Theorem of external division of chords)	
	∴ PT × 8 = 16 × 6 ...[From (i) and given]	½
	∴ PT = $\frac{16 \times 6}{8}$	
	∴ PT = 12 units ...(ii)	
	∴ PT = PS + TS ...(P – S – T)	
	∴ 12 = 8 + TS	
	∴ TS = 12 – 8	
	∴ **TS = 4 units**	½
A.4.	**Solve the following questions : (Any 3)**	
(1)	You know that Slope of line = $\frac{y_2 - y_1}{x_2 - x_1}$	½
	Slope of line AB = $\frac{2-1}{8-6} = \frac{1}{2}$ .... (I)	
	Slope of line BC = $\frac{4-2}{9-8}$ = 2............. (II)	½
	Slope of line CD = $\frac{3-4}{7-9} = \frac{1}{2}$ .... (III)	
	Slope of line DA = $\frac{3-1}{7-6}$ = 2 .... (IV)	½
	Slope of line AB = Slope of line CD .... From (I) and (II)	
	∴ line AB || line CD	
	Slope of line BC = Slope of line DA .... From (II) and (IV)	½
	∴ line BC || line DA	
	Both the pairs of opposite sides of the quadrilateral are parallel	½
	Slope of line AB = Slope of line CD .... From (I) and (II)	
	∴ ☐ABCD is a parallelogram.	½

(2) Slope of line AB = $\frac{-0.5-2}{1-0} = \frac{-2.5}{1}$ 1

$\therefore$ Slope of line AB = –2.5 ...(i)

Slope of line AC = $\frac{-3-2}{2-0} = \frac{-5}{2}$

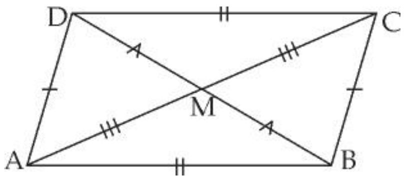
$\therefore$ Slope of line AC = –2.5 ...(ii) 1

$\therefore$ Slope of line AB = Slope of line AC ...[From (i) and (ii)] ½

Also, they have a common point A.

$\therefore$ **Points A, B and C are collinear points.** ½

(3) □ABCD is a parallelogram ...(Given)



½

$\therefore$ BM = $\frac{1}{2}$ BD ...(i) ½

AM = $\frac{1}{2}$ AC ...(ii)

[Diagonals of a parallelogram bisect each other] ½

$\therefore$ AM = $\frac{1}{2} \times 14$ = 7cm ½

In ΔABC, seg BM is the a median ...[From (ii) and Definition]

$\therefore$ $AB^2 + BC^2 = 2AM^2 + 2BM^2$ ...(Apollonius theorem) ½

$130 = 2(7^2 + BM^2)$

$\therefore$ $\frac{130}{2} = 49 + BM^2$

$\therefore$ $65 - 49 = BM^2$

$\therefore$ $BM^2 = 16$ ½

$\therefore$ BM = 4 cm ...(Taking square roots)

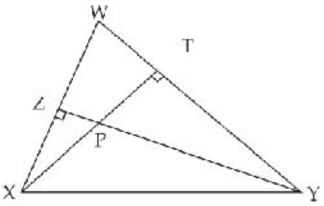
$\therefore$ $\frac{1}{2}$BD = 4cm ...[From (i)]

$\therefore$ **BD = 8 cm** ½

(4) To Prove: ½

(i) □WZPT is a cyclic quadrilateral

(ii) Points X, Z, T and Y are concyclic points.



Proof :

∠XTW = 90° ...(i) (Given) ½

∠YZW = 90° ...(ii) (Given)

∠XTW + ∠YZW = 90° + 90° ...(Adding (i) and (ii)) ½

∴ ∠PTW +∠PZW = 180° (X – P – T, Y – P – Z)

□WZPT is a cyclic quadrilateral...(Converse of cyclic quadrilateral theorem)

∠XZY = ∠XTY = 90° ...(Given)

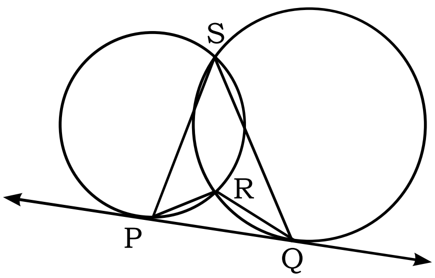
∴ ∠XZY = ∠XTY ½

∴ seg XY subtends congruent angle at points Z and T which are on the same side of line XY.

∴ **Point X, Z, T and Y are concyclic points.** ½

A.5. Solve the following questions : (Any 1)

(1) 2



Construction: Draw seg RS

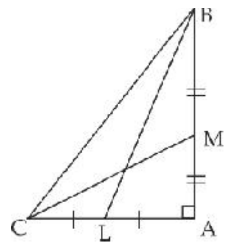
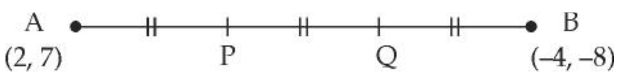
Proof: ∠PSR ≅ ∠RPQ ...(i)
∠QSR ≅ ∠RQP ...(ii)
} (Angles in alternate segments) ½

In ΔPRQ,

∠PRQ + ∠RPQ + ∠RQP = 180° ...(sum of angles of a triangle is 180°) ½

∴ ∠PRQ + ∠PSR + ∠QSR = 180° ...[From (i) and (ii)]

∴ ∠PRQ + ∠PSQ = 180° ...(Angle addition property) ½

(2)	 	½
	To Prove :	
	$4(BL^2 + CM^2) = 5BC^2$	½
	Proof :	
	In ΔBAC, $\angle BAC = 90^\circ$ ...(Given)	
	$\therefore$ $BC^2 = AB^2 + AC^2$ ...(i) (By Pythagoras theorem)	½
	In ΔBAL, $\angle BAC = 90^\circ$ ...(Given)	
	$\therefore$ $BL^2 = AB^2 + AL^2$ ...(ii) (By Pythagoras theorem)	½
	In ΔCAM, $\angle CAM = 90^\circ$ ...(Given)	
	$\therefore$ $CM^2 = AC^2 + AM^2$ ...(iii) (By Pythagoras theorem)	
	Adding (ii) and (iii),	
	$BL^2 + CM^2 = AB^2 + AL^2 + AC^2 + AM^2$	½
	$\therefore$ $BL^2 + CM^2 = AB^2 + AC^2 + AL^2 + AM^2$	
	$\therefore$ $BL^2 + CM^2 = BC^2 + AL^2 + AM^2$ [From (i)]	½
	$\therefore$ $BL^2 + CM^2 = BC^2 + \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}AB\right)^2$ [$\because$ L and M are the midpoint of sides AC and AB respectively]	
	$\therefore$ $BL^2 + CM^2 = BC^2 + \frac{AC^2}{4} + \frac{AB^2}{4}$	½
	$\therefore$ $4(BL^2 + CM^2) = 4BC^2 + AC^2 + AB^2$ (Multiplying throughout by 4)	
	$\therefore$ $4(BL^2 + CM^2) = 4BC^2 + BC^2$ ...[From (i)]	
	$\therefore$ $\mathbf{4(BL^2 + CM^2) = 5BC^2}$	½
A.6.	**Solve the following questions : (Any 1)**	
(1)	 	½
	Let point P and Q be two points which divide seg AB in three equal parts.	

Point P divides seg AB in the ratio 1 : 2

By Section formula,

$$P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$ ½

$$P\left(\frac{1\times(-4)+2\times 2}{1+2}, \frac{1\times(-8)+2\times 7}{1+2}\right)$$ ½

$\therefore$ $$P\left(\frac{-4+4}{3}, \frac{-8+14}{3}\right)$$

$\therefore$ $$P\left(\frac{0}{3}, \frac{6}{3}\right)$$ ½

$\therefore$ P (0, 2)

Also, PQ = QB

$\therefore$ Point Q is midpoint of seg PB.

By midpoint formula,

$\therefore$ $$Q\left(\frac{0+(-4)}{2}; \frac{2+(-8)}{2}\right)$$ ½

$\therefore$ $$Q\left(\frac{-4}{2}, \frac{-6}{2}\right)$$

$\therefore$ Q (–2, –3)

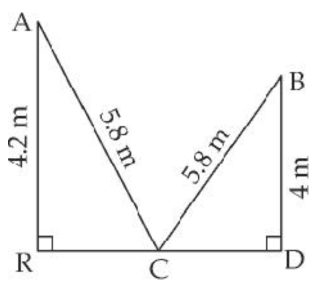
$\therefore$ **P(0, 2) and Q(–2, –3) are points which trisects seg AB** ½

(2) Let RD represents the width of the street.

BD represents the first building.

AR represents the second building

CA and CB are two different positions of the same ladder from point C.



AR = 4.2 m, BD = 4 m, AC = BC = 5.8 m, RD = ?

In ΔARC, $\angle$R = 90° ...(Given)

$\therefore$ $AC^2 = AR^2 + CR^2$ ...(By Pythagoras theorem)

$\therefore$ $(5.8)^2 = (4.2)^2 + CR^2$

$\therefore$ $CR^2 = (5.8)^2 - (4.2)^2$

$\therefore$ $CR^2 = (5.8 + 4.2)(5.8 - 4.2)$

$\therefore$ $CR^2 = 10 \times 1.6$

$\therefore$ $CR^2 = 16$

$\therefore$ $CR = 4$ m ...(Taking square root)

In ΔBDC, $\angle D = 90^\circ$ ...(Given)

$\therefore$ $BC^2 = CD^2 + BD^2$ ...(By Pythagoras theorem)

$\therefore$ $5.8^2 = CD^2 + 4^2$

$\therefore$ $CD^2 = (5.8)^2 - 4^2$

$\therefore$ $CD^2 = (5.8 + 4)(5.8 - 4)$

$\therefore$ $CD^2 = 9.8 \times 1.8$

$\therefore$ $CD^2 = \frac{98}{10} \times \frac{18}{10}$

$\therefore$ $CD^2 = \frac{98 \times 18}{100}$

$\therefore$ $CD^2 = \frac{98 \times 2 \times 9}{100}$

$\therefore$ $CD^2 = \frac{196 \times 9}{100}$

$\therefore$ $CD = \frac{14 \times 3}{10}$ ...(Taking square roots)

$\therefore$ $CD = \frac{42}{10}$

$\therefore$ $CD = 4.2$ m

RD = RC + CD ...(R – C – D)

= 4 + 4.2

RD = 8.2 m

$\therefore$ **Width of the street is 8.2 m**