MT

2018 ___ ___ 1100

MT - GEOMETRY - SEMI PRELIM - I : PAPER - 1

Time : 2 Hours **(Model Answer Paper)** **Max. Marks : 40**

A.1. (A) Solve the following : (Any 4) **1**

(1) Perimeter of a parallelogram = 2 (sum of lengths of adjacent sides) ½

= 2 (3 + 4) =14 cm ½

∴ Perimeter of a parallelogram is 14 cm

(2) Longest chord of a circle is Diameter ½

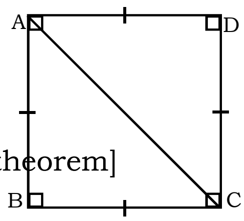
Diameter = 2 × radius

= 2 × 8

= 16

∴ Length of longest chord of a circle is 16 cm. ½

(3) Let □ABCD be a square



AB = BC = 5 cm

In ΔABC,

$\angle ABC = 90^\circ$ (Angle of a square)

$AC^2 = AB^2 + BC^2$ [Pythagoras theorem] ½

$= 5^2 + 5^2$

$= 25 + 25$

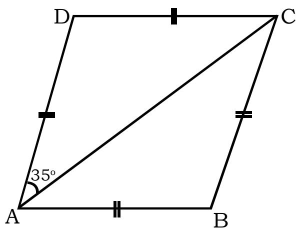
∴ $AC^2 = 50$

∴ $AC = \sqrt{50} = \sqrt{25 \times 2}$

∴ $AC = 5\sqrt{2}$

Length of the diagonal is $5\sqrt{2}$ cm ½

(4)



□ABCD is a rhombus

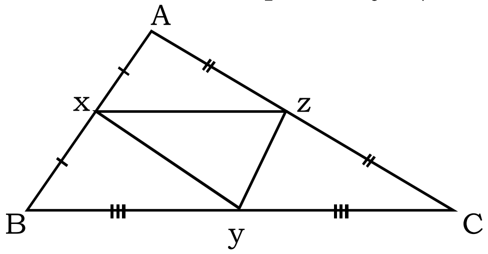
In ΔDAC,
side AD $\cong$ side DC (sides of a rhombus)
$\therefore$ $\angle DAC \cong \angle DCA$ (Isosceles triangle theorem) ½
$\therefore$ $\angle DAC = \angle DCA = 35^\circ$
$\therefore$ $\angle ADC = 110^\circ$ (Remaining angle)
$\angle ADC \cong \angle ABC$ (opp. angles of a rhombus are congruent) ½
$\therefore$ $\angle ABC = 110^\circ$

(5) Equation of x-axis is '$y = 0$'. 1

(6) Equation of a line passing through '–4' on x-axis and parallel to y axis is $x = -4$ 1

A.1. (B) Solve the following : (Any 2)

(1) AB = 5 cm, AC = 9 cm and BC = 11 cm (Given) ½
In ΔABC,
X and Y are the midpoints of sides AB and BC respectively. (Given)



$\therefore$ $XY = \frac{1}{2}$ AC (Midpoint theorem) ½
$\therefore$ $XY = \frac{1}{2} \times 9$
$\therefore$ XY = 4.5 cm
In ΔABC,
Y and Z are the midpoints of sides BC and AC respectively. (Given)
$\therefore$ $YZ = \frac{1}{2}$ AB (Midpoint theorem) ½
$\therefore$ $YZ = \frac{1}{2} \times 5$
$\therefore$ YZ = 2.5 cm
In ΔABC,
X and Z are the midpoints of sides AB and AC respectively. (Given)
$\therefore$ XZ = — BC (Midpoint theorem) ½
$\therefore$ $XZ = \frac{1}{2} \times 11$
$\therefore$ XZ = 5.5 cm
XY = 4.5 cm, YZ = 2.5 cm XZ = 5.5 cm.

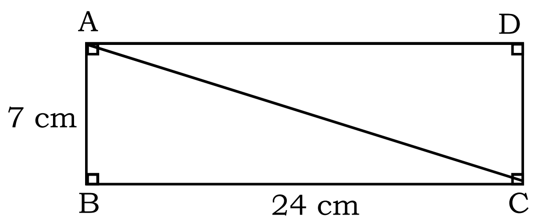
(2) Let the centre of the circle be O
Let seg OM $\perp$ chord AB such that A-M-B

$AM = \frac{1}{2} AB$ (Perpendicular drawn from the centre of the circle bisects the chord) ½

$\therefore AM = \frac{1}{2} \times 12$

$\therefore AM = 6$ cm
Draw seg OA
In right angled ΔOMA,
$OA^2 = OM^2 + AM^2$ (by Pythagoras' theorem) ½
$\therefore OA^2 = 8^2 + 6^2$
$\therefore OA^2 = 64 + 36$
$\therefore OA^2 = 100$
$\therefore OA = \sqrt{100}$
$\therefore OA = 10$ cm ½
$\therefore$ radius of the circle is 10 cm
Diameter of the circle is twice the radius
$\therefore$ diameter $= 2 \times 10$
$\therefore$ diameter $= 20$ cm
Diameter of the circle is 20 cm. ½

(3)



Let □ABCD be a given rectangle. ½
AB = 7 cm, BC = 24 cm
In ΔABC,
$\angle ABC = 90^\circ$ ....(Angle of a rectangle) ½
By Pythagoras theorem,
$AC^2 = AB^2 + BC^2$ $\therefore AC^2 = 7^2 + 24^2$
$\therefore AC^2 = 49 + 576$
$\therefore AC^2 = 625$
$\therefore AC = \sqrt{625}$ $\therefore AC = 25$ cm 1

The length of the diagonal is 25 cm. ½

A.2. (A) Solve the following MCQs :

(1) Altitude on the hypotenuse of a right angled triangle divides it in two parts of lengths 4 cm and 9 cm. Find the length of the altitude.
6 cm — 1

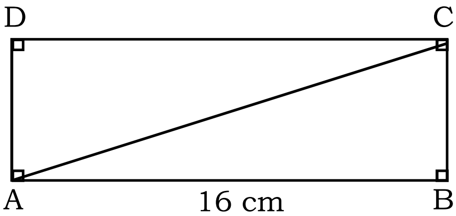
(2) 7 cm — 1

(3) (C) 5 — 1

(4) (B) (3, 4, 5) — 1

A.2. (B) Solve the following : (Any 2)

(1)



□ABCD is a rectangle — ½

A(□ABCD) = length × breadth

∴ 192 = AB × BC

∴ 192 = 16 × BC

$\frac{192}{16}$ = BC

∴ BC = 12 cm — ½

In ΔABC, ∠ABC = 90° ...(Angle of a rectangle)

∴ $AC^2 = AB^2 + BC^2$ ...(By Pythagoras theorem) — ½

$AC^2 = (16)^2 + (12)^2 = 256 + 144$

∴ $AC^2 = 400$

∴ AC = 20 cm ...(Taking square roots)

∴ **length of the diagonal is 20 cm** — ½

(2) □MRPN is a cyclic quadrilateral ...(Given)

∴ ∠R + ∠N = 180° ...(Cyclic quadrilateral theorem) — ½

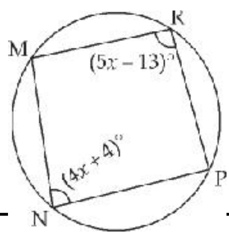
∴ $5x - 13 + 4x + 4 = 180$

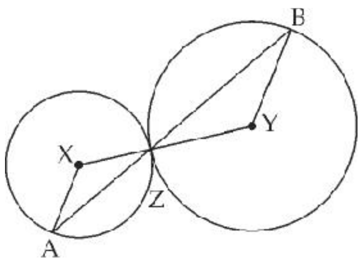
∴ $9x - 9 = 180$

∴ $9x = 180 + 9$

∴ $9x = 189$

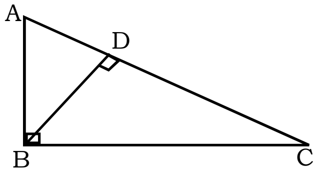
∴ $x = \frac{189}{9}$ — ½



	∴ $x = 21$	½
	∴ $m\angle R = 5x - 13 = 5 \times 21 - 13 = 105 - 13 = 92°$	
	∴ $m\angle N = 4x + 4 = 4 \times 21 + 4 = 84 + 4 = 88$	½
(3)	$B(k, -5) = (x_1, y_1)$ $C(1, 2) = (x_2, y_2)$	
	Slope of line BC = $\frac{y_2 - y_1}{x_2 - x_1}$	½
	∴ $7 = \frac{2-(-5)}{1-k}$	½
	∴ $7(1 - k) = 2 + 5$ ∴ $7(1 - k) = 7$	
	∴ $1 - k = \frac{7}{7}$ ∴ $1 - k = 1$	½
	∴ $1 - 1 = k$ ∴ **$k = 0$**	½
A.3. (1)	**(A) Solve the following activity : (Any 2)**	
		
	Construction : Draw segments XZ and YZ Proof : By theorem of touching circles, points X, Z, Y are collinear points $\angle XZA \cong \angle BZY$ (Vertically Opposite angles) Let $\angle XZA = \angle BZY = a$ ...(i) seg XA ≅ seg XZ Radii of the same circle ∴ $\angle XAZ = \angle YBZ = a$ ...(ii) (Isosceles triangle theorem) seg YB ≅ YZ Radii of the same circle ∴ $\angle BZY = \angle YBZ = a$ ...(iii) (Isosceles triangle theorem) $m\angle XAZ = m\angle YBZ = a$ ...[From (i), (ii) and (iii)]	

∴ Radius XA || radius YB [alternate angles test]

(2) (1) In ΔABC, ∠ABC = 90°
(2) seg BD ⊥ hypotenuse AC, A - D - C



To Prove :
ΔABC ~ ΔADB ~ ΔBDC

Proof :

In ΔABC and ΔADB,

∠ABC ≅ ∠ADB [Each is a right angle]

∠A ≅ ∠A [Common angle]

∴ [ΔABC] ~ [ΔADB] ...(i) (By AA Test of similarity)

In ΔABC and ΔBDC,

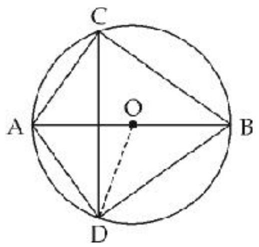
[∠ABC] ≅ [∠BDC] (Each is a right angle)

[∠C] ≅ [∠C] (Common angle)

∴ ΔABC ~ ΔBDC ...(ii) [By AA Test of similarity]

∴ ΔABC ~ [ΔADB] ~ [∠BDC] ...[From (i) and (ii)]

(3)



Proof : Draw seg OD.

∠ACB = [90°] (∵ Angle inscribed in a semicircle)

∠DCB = [45°] (∵ CD bisects ∠ACB)

m(arc DB) = [90°] (Inscribed angle theorem)

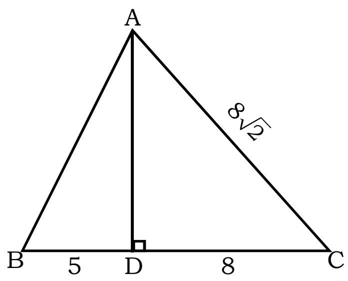
∠DOB = [90°] ...(i) (Definition of measure of an arc)

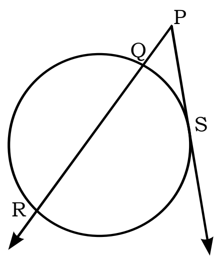
seg OA ≅ seg OB ...(ii) [Radii of same circle]

seg OD is [Perpendicular bisector] of seg AB [From (i) and (ii)]

∴ seg AD ≅ seg BD [Perpendicular bisector theorem]

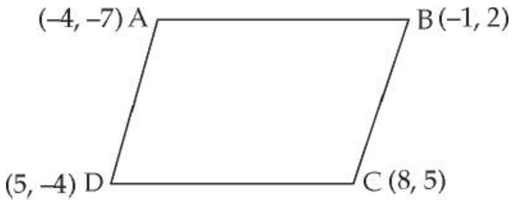
A.3.	**(B) Solve the following : (Any 2)**	
(1)	L(x, 7) and M(1, 15)	
	By distance formula,	
	d(L, M) = $\sqrt{(x-1)^2+(7-15)^2}$	½
	$\therefore$ 10 = $\sqrt{(x-1)^2+(-8)^2}$	
	Squaring both the sides we get,	
	100 = $(x-1)^2 + 64$	½
	$\therefore$ 100 – 64 = $(x-1)^2$	
	$\therefore$ $(x-1)^2 = 36$	
	$\therefore$ $x - 1 = \pm 6$ (Taking square roots)	½
	$\therefore$ $x - 1 = 6$ or $x - 1 = -6$	
	$\therefore$ $x = 6 + 1$ or $x = -6 + 1$	
	$\therefore$ $x = 7$ or $x = -5$	
	$\therefore$ **$x = 7$ or $x = -5$**	½
(2)	In ΔADC	
	$\angle$ADC = 90°, $\angle$C = 45°,	
	$\therefore$ $\angle$DAC = 45°	
	AD = DC = $\frac{1}{\sqrt{2}} \times 8\sqrt{2}$	½
	by 45° – 45° – 90° theorem	
	DC = 8 $\therefore$ AD = 8	
	BC = BD + DC = 5 + 8	
	BC = 13	½
(3)	PS^2 = PQ × PR (Tangent secant segments theorem)	½
	= PQ × (PQ + QR)	
	= 3.6 × [3.6 + 6.4]	
	= 3.6 × 10	
	= 36	
	$\therefore$ PS = 6	½





A.4. Solve the following questions : (Any 3)

(1)



Slope of AB $= \frac{2-(-7)}{-1-(-4)}$

$= \frac{2+7}{-1+4}$

$= \frac{9}{3}$

Slope of AB $= 3$ ...(i) ½

Slope of BC $= \frac{5-2}{8-(-1)}$

$= \frac{3}{8+1}$

$= \frac{3}{9}$

Slope of BC $= \frac{1}{3}$ ...(ii) ½

Slope of AD $= \frac{-4-(-7)}{5-(-4)}$

$= \frac{-4+7}{5+4}$

$= \frac{3}{9}$

Slope of AD $= \frac{1}{3}$ ...(iii) ½

Slope of CD $= \frac{-4-5}{5-8}$

$= \frac{-9}{-3}$

Slope of CD = 3 ...(iv) ½

Slope of line AB = Slope of line CD [From (i) and (iv)]

$\therefore$ Line AB || Line CD ...(v)

$\therefore$ Slope of line BC = Slope of line AD [From (ii) and (iii)] ½

$\therefore$ Line BC || Line AD ...(vi)

In □ABCD, AB || CD ...[From (v)]

BC || AD ...[From (vi)]

$\therefore$ **□ABCD is a parallelogram.** (Definition) ½

(2) Slope of line PQ $= \dfrac{\frac{8}{5} - 2}{2 - 1}$ ½

$= \dfrac{\frac{8 - 10}{5}}{1}$

$\therefore$ Slope of line PQ $= \dfrac{-2}{5}$ ...(i) ½

Slope of line QR $= \dfrac{\frac{6}{5} - \frac{8}{5}}{3 - 2}$ ½

$= \dfrac{\frac{6 - 8}{5}}{1}$

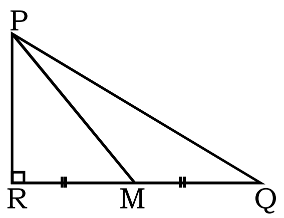
$\therefore$ Slope of line QR $= \dfrac{-2}{5}$ ...(ii) ½

$\therefore$ Slope of line PQ = Slope of line QR [From (i) and (ii)]

Also, they have a common point Q.

$\therefore$ **Points P, Q and R are collinear points.** ½

(3)



Proof :

In ΔPRQ, ∠PRQ = 90° ...(Given) ½

∴ $PQ^2 = PR^2 + QR^2$ ...(i) (By Pythagoras theorem)

∴ QR = 2RM ...(ii) (M is the midpoint of seg QR)

∴ $PQ^2 = PR^2 + (2\ RM)^2$ ...[From (i) and (ii)] ½

∴ $PQ^2 = PR^2 + 4\ RM^2$ ...(iii) ½

In ΔPRM, ∠PRM = 90°

∴ $PM^2 = PR^2 + RM^2$ ...(Pythagoras theorem) ½

∴ $RM^2 = PM^2 - PR^2$ ...(iv)

∴ $PQ^2 = PR^2 + 4\ (PM^2 - PR^2)$ ...[From (iii) and (iv)] ½

∴ $PQ^2 = PR^2 + 4\ PM^2 - 4\ PR^2$

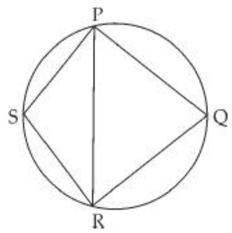
∴ $\boxed{PQ^2 = 4\ PM^2 - 3\ PR^2}$ ½

(4) □PQRS is a cyclic quadrilateral (Given) ½

∴ ∠PQR + ∠PSR = 180° (Cyclic quadrilateral theorem) ½

∴ ∠PQR + 110° = 180°

∴ ∠PQR= 180° – 110° ...(i) ½



m∠PQR = 70°

$\angle PSR = \frac{1}{2}$ m(arc PQR)

(Inscribed angle theorem)

∴ $110° = \frac{1}{2}$ m(arc PQR) ... (v) [From (iv)]

∴ **m(arc PQR) = 220°** ...(ii) ½

∴ In ΔPQR, side PQ ≅ side RQ (Given)

∴ ∠QPR ≅ ∠QPR ...(iii) (Isosceles triangle theorem)

In ΔPQR,

∠PQR + ∠PRQ + ∠QPR = 180° (Sum of all angles of a triangle is 180°)

∴ 70° + ∠QPR + ∠QPR = 180° (From (i) and (ii)]

∴ 2∠QPR = 180° – 70°

∴ 2∠QPR = 110°

∴ ∠QPR = 55° ...(iv) ½

∠QPR = $\frac{1}{2}$ m(arc QR) (Inscribed angle theorem)

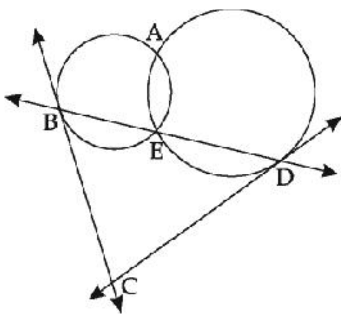
∴ 55 = $\frac{1}{2}$ × m(arc QR) [From (iv)]

∴ **m(arc QR) = 110°** [From (iii) and (iv)]

∴ **∠PRQ = 55°** ½

A.5. Solve the following questions : (Any 1)

(1)



Proof: ∠BAE = ∠EBC ...(i)
∠DAE ≅ ∠EDC ...(ii) } (Angles in alternate segments) 1 ½
In ΔBCD,
∠BCD + ∠DBC + ∠BDC = 180° Sum of angle of atriangle is 180°) ½
∴ ∠BCD + ∠EBC + ∠EDC = 180° (B–E–D) ½
∠BCD + ∠BAE + ∠DAE = 180° [From (i) & (ii)] ½
∠BCD + ∠BAD = 180° (Angle addition property) ½
∴ □ABCD is a cyclic (Converse of cyclic) ½

(2) PQ = QR = PR ...(i) [sides of an equilateral triangle] ½
In ΔPTS, ∠PTS = 90° (Construction)
∴ $PS^2 = PT^2 + ST^2$ ...(ii) (By Pythagoras theorem) ½
In ΔPTQ,
∠PTQ = 90° (Construction)
∠PQT = 60° (angle of an equilated triangle) ½
∠QPT = 30° (remaining angle) ½
∠ ΔPTQ is a 30° – 60° – 90° triangle
By 30° – 60° – 90° triangle theorem,

$\therefore$ PT = $\frac{\sqrt{3}}{2}$ PQ ...(iii) (side opposite to 60°)

$\therefore$ QT = $\frac{1}{2}$ PQ ...(iv) (Side opposite to 30°)

ST = QT – QS (Q – S – T) ½

$\therefore$ ST = $\frac{1}{2}$ PQ – $\frac{1}{3}$ QR [From (iv) and given]

$\therefore$ ST = $\frac{1}{2}$ PQ – $\frac{1}{3}$ PQ [From (i)]

$\therefore$ ST = $\frac{3PQ - 2PQ}{6}$ ½

$\therefore$ ST = $\frac{1}{6}$ PQ ...(v)

$\therefore$ $PS^2 = \left(\frac{\sqrt{3}}{2}PQ\right)^2 + \left(\frac{PQ}{6}\right)^2$ [From (ii) (iii) and (v)]

$\therefore$ $PS^2 = \frac{3PQ^2}{4} + \frac{PQ^2}{36}$ ½

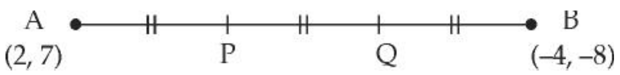
$\therefore$ $PS^2 = \frac{27PQ^2 + PQ^2}{36}$

$\therefore$ $PS^2 = \frac{28PQ^2}{36}$

$\therefore$ $PS^2 = \frac{7}{9}PQ^2$

$\therefore$ **$9\ PS^2 = 7\ PQ^2$** ½

A.6. Solve the following questions : (Any 1)

(1) 

½

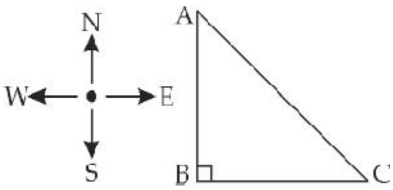
Let point P and Q be two points which divide seg AB in three equal parts.

Point P divides seg AB in the ratio 1 : 2

By Section formula,

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$ ½

$$P\left(\frac{1\times(-4) + 2\times 2}{1+2}, \frac{1\times(-8) + 2\times 7}{1+2}\right)$$ ½

	$\therefore$ P $\left(\frac{-4+4}{3}, \frac{-8+14}{3}\right)$	
	$\therefore$ P $\left(\frac{0}{3}, \frac{6}{3}\right)$	½
	$\therefore$ P (0, 2)	
	Also, PQ = QB	
	$\therefore$ Point Q is midpoint of seg PB.	
	By midpoint formula,	
	$\therefore$ Q $\left(\frac{0+(-4)}{2}; \frac{2+(-8)}{2}\right)$	½
	$\therefore$ Q $\left(\frac{-4}{2}, \frac{-6}{2}\right)$	
	$\therefore$ Q (–2, –3)	
	$\therefore$ **P(0, 2) and Q(–2, –3) are points which trisects seg AB**	½
(2)		
	B represents starting point of journey.	
	BA is the distance travelled by Prasad in North direction.	
	BC is the distance travelled by Pranali in east direction.	1
	AC is the distance between Pranali and Prasad after two hours.	
	Let the speed of each one be x km/hr.	
	$\therefore$ Distance travelled by each one hour is $2x$ km.	½
	i.e. AB = BC = $2x$ km	
	In ΔABC, $\angle$B = 90° ...(Line joining adjacent direction are $\perp$ to each other)	
	$\therefore$ $AB^2 + BC^2 = AC^2$ ...(By Pythagoras theorem)	½
	$\therefore$ $(2x)^2 + (2x)^2 = (15\sqrt{2})^2$	

$\therefore \quad 4x^2 + 4x^2 = 225 \times 2$

$\therefore \quad 8x^2 = 225 \times 2$ ½

$\therefore \quad x^2 = \frac{225 \times 2}{8}$

$\therefore \quad x^2 = \frac{225}{4}$

$\therefore \quad x = \frac{15}{2}$ ...(Taking square roots)

$\therefore \quad x = 7.5$

$\therefore$ **Speed of each one is 7.5 km / hr** ½